Oyster River Cooperative School District

K-12 Mathematics Curriculum
WORKING DRAFT
2009-2010

Mathematics Curriculum / Assessment Committee 2009-2010

Mast Way School

Trisha Hall	Kindergarten
Whitney Burke	Grade 1
Barbra Bertrand	Grade 2
Barbara Paquette	Grade 3
Deb McNelly	Grade 4
Pat Lutz	Special Education

Moharimet School

Jessica Raspa	Kindergarten
Berneen Bratt	Grade 1
Terry Nadeau	Grade 2
Liz Schmitt	Grade 3

Middle School

Bill Sullivan	Grade 5
Deb Lachance	Grade 6
Michelle McInnes	Grade 7
Lisa Miller	Grade 8

High School

Brian O'Connell	Math Teacher

District

Danielle Bolduc	Director of Instruction

:

State of New Hampshire
K-12 Mathematics Curriculum Framework

STRAND DESCRIPTORS

Content Strands

Strand 1: Number and Operations

Purpose: Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts.

Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

Strand 2: Geometry and Measurement

Purpose: Geometry and the related area of measurement help students represent, describe, and make sense of the world in which they live. Geometry is also a natural place for students to develop their reasoning and justification skills.

We live in a three-dimensional world. To interpret, understand, and appreciate that world, students need to develop an understanding of space. In addition, success in mathematics depends, in part, on the development of spatial abilities. Spatial skills include making and interpreting drawings, forming mental images, and visualizing changes.

Measurement is the process of assigning a numerical value to an attribute of an object. The study of measurement provides students with techniques and tools they will need to describe and analyze their

world. It also provides an opportunity to make connections within mathematics and between mathematics and other curricular areas. High school students must develop more mature insights into the essential role of measurement as a link between the abstractness of mathematics and the concreteness of the real-world.

In both areas, geometry and measurement, students need to investigate, experiment, and explore geometric properties using both technology and hands-on materials.

Strand 3: Functions and Algebra

Purpose: Algebra is the language through which much of mathematics is communicated. Students in Kindergarten begin to explore algebraic concepts using informal representations (e.g., words, physical models, tables, graphs). In later years students progress to more abstract representations. The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions. Students at all grade-levels should recognize, describe, and generalize patterns and build mathematical models to describe, interpret, and predict the behavior of real-world phenomenon. Algebraic processes are important tools that students can use throughout their lives.

Strand 4: Data, Statistics, and Probability

Purpose: Collecting, organizing, and displaying data, as well as interpreting and analyzing the information to make decisions and predictions, have become very important in our society. Statistical instruction should be carried out in a spirit of investigation and exploration so students can answer and formulate questions about data. Probability should be studied in familiar contexts. Students need to investigate fairness, chances of winning, and uncertainty. Technology should be used as a tool throughout the investigation process.

Process Strands

Since it is crucial that process standards (problem-solving, reasoning, proof, communication, connections, and representations) are not seen separate from content standards the process standards have been imbedded throughout the Number and Operations, Geometry and Measurement, Functions and Algebra, and Data, Statistics, and Probability strands (e.g., M:F&A:4:1 **Identifies and extends to specific cases a variety of patterns** (linear and nonlinear) represented in models, tables or sequences; and writes a rule in words or^{sc} symbols to find the next case.). This mirrors classroom instruction as in most classes, as students are learning content knowledge, instruction is also focusing on improving their abilities in problem solving, reasoning, and communication; furthermore, students are looking for and making appropriate connections, and they are able to understand and use multiple representations of mathematical ideas. Since it is crucial that students are strong in both content and process knowledge, we have included two local process strands. These process strands are in addition to the process standards that are embedded in the content standards and are included to help guide local curriculum, assessment, and instruction. The process standards have been separated by grade-spans (K–2, 3–5, 6–8, and 9–12). Each span should be thought of as building upon the skills and concepts in the previous span.

Strand 5: Problem Solving, Reasoning, and Proof

Purpose: Problem solving should serve as the organizing feature of the mathematics curriculum as well as other areas of study and be applied to everyday activities. Thus, problem solving approaches should be used to investigate and understand new mathematical content, with students working sometimes independently and sometimes in groups. Students should have many experiences in posing and solving problems from their world, from data that are meaningful to them, and from mathematical investigations. Students should build a positive disposition toward problem solving, including the confidence needed to explore unique problems and increasingly complex tasks.

Strand 6: Communication, Connections, and Representations

Purpose: Reading, writing, talking, listening, and modeling provide students with the opportunity to develop deeper mathematical understanding and to integrate the language of mathematics into their world. Actively exploring, investigating, describing, and explaining mathematical ideas promote communication which leads to a greater comprehension of mathematical concepts.

Representing ideas and connecting the representations to mathematics lies at the heart of understanding mathematics. Representations make mathematical ideas more concrete and available for reflection, and they help students recognize the common mathematical nature of different situations. Students can develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations.

Mathematical topics, ideas, and procedures must be connected to each other and to the students' everyday experiences, both in and out of school. In particular, mathematics must be connected to all other curriculum areas. Mathematical connections will help students become aware of the usefulness of mathematics, serve to bridge the concrete and the abstract, and enable deeper understanding of important ideas.

New Hampshire K-12 Mathematics Curriculum Frameworks,

K-8 Grade Level Expectations and High School Grade Span Expectations can be found at:

http://www.ed.state.nh.us/education/doe/organization/curriculum/NECAP/GLEs.htm.

The High School Mathematics Department has developed Mathematics Competencies for each subject as required by the New Hampshire Department of Education Administrative Rules (Ed 306.27). The competencies are saved on the shared folder in the high school and will be included here in the final document. New Hampshire High School Mathematics Grade-Span Expectations can be viewed at:

http://www.education.nh.gov/instruction/assessment/necap/documents/math_gse.pdf

Standards and Proficiencies by Grade Level

Kindergarten

Content Strands

Strand 1- Number and Operations

1. **Demonstrates conceptual understanding of rational numbers through the use of models, explanations, or other representations with respect to:**

    - recognizing and sequencing numbers 0 to 12
    - counting by rote 0 to 20
    - practicing formation of numbers 0 to 12
    - making, counting, and analyzing sets
    - investigating fractions as equal shares (whole and one-half)

2. **Demonstrates an understanding of the relative magnitude of numbers from 0 -20 using models, representation, or number lines by:**

    - counting with one-to-one correspondence from 0 to 20
    - investigating the quantity associated with numerals 0-20
    - investigating counting patterns (ex. – using a hundreds chart to show patterns)
    - grouping objects into sets of tens and ones (investigating place value)
    - comparing whole numbers to each other or to benchmark whole numbers (5, 10)
    - comparing amounts using mathematical language (for example: same, different, more, less and equal)

3. **Using concrete materials, demonstrates conceptual understanding of mathematical operations by:**

    - investigating addition and subtraction of whole numbers (from 0 to 10) by solving problems involving joining actions, separating actions, part-part whole relationships, and comparison situations: (See Appendix A in NH State Frameworks)
    - investigating addition of more than two one-digit whole numbers.
    - developing a recognition of symbols for addition, subtraction and equality

4. No standard at this grade level

5. Demonstrates an understanding of monetary value by:

- playing constructively with coins
- knowing the names and values of coins (penny, nickel and dime).

6. Mentally adds and subtracts whole numbers by:

- naming the number that is one more than the original number.
- naming the number that is one less than the original number.

    (*The intent of this proficiency is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.*)

7. Demonstrates an understanding of estimation by:

- estimating the number of objects in a set up to 20
- revising estimates as objects are counted (e.g., A student estimates the number of pennies in a jar as 20. Then the student counts the first 10 and makes another estimate based on those that have been counted and those that remain in the jar.)

8. No standard at this grade level.

Strand 2- Geometry and Measurement

1. Demonstrates conceptual understanding of two-dimensional shapes by:

- sorting or classifying polygons (triangles, squares, rectangles, rhombi, trapezoids, and hexagons) using one non-measurable or measurable attribute
- recognizing and naming polygons and circles in the environment
- copying and building polygons and circles using concrete materials
- investigating and observing results of combining shapes

2. No standard at this grade level

3. No standard at this grade level

4. No standard at this grade level

5. No standard at this grade level

6. No standard at this grade level

7. Demonstrates conceptual understanding of measurement by:

- using comparative language to describe and compare attributes of objects (length – *longer, shorter*, height – *taller, shorter,* weight – *heavier, lighter*, temperature – *warmer, cooler,* and capacity – *more, less*)
- comparing objects visually and with direct comparison
- practicing non-standard measurement with concrete materials

8. Demonstrates an understanding of elapsed and accrued time by:

- investigating calendar patterns (days of the week, yesterday, today, and tomorrow)
- recognizing the passage of time throughout the day and following the sequence of events in a day
- identifying a clock and calendar as measurement tools (days of week, months of year).

9. Demonstrates an understanding of spatial relationships using location and position by:

- using positional words (ex. – over, under, behind, in front of, next to, in between) to locate and describe where an object is found in the environment.

10. No standard at this grade level.

Strand 3- Functions and Algebra

1. Identifies and extends a variety of patterns (sequences of shapes, sounds, movement, color, and letters) by:

- extending the pattern to the next one, two, or three elements
- translating simple patterns across formats (e.g., an ABB pattern can be represented by snap, clap, clap or red, yellow, yellow)
- identifying number patterns in the environment

2. No standard at this grade level

3. No standard at this grade level

4. Develops the concept of equality through:

- comparisons of concrete materials, graph data, and whole numbers.
- investigating known and unknown numbers in simple number stories (e.g. "We have 3 straws. How many more until we get to 10?")

Strand 4- Data, Statistics, and Probability

1. Interprets a given representation created by the class (models and tally charts) by:

- constructing simple graphs
- collecting real-life information to make graphs
- counting, sorting, and categorizing data or objects
- analyzing concrete data using words, diagrams, or verbal/scribed responses to express answers (e.g. "How many rainy days in March?")

2. Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using more, less, or equal (e.g. have there been more, less, or the same number of cloudy days compared to sunny days this week?)

3. No standard at this grade level

4. No standard at this grade level

5. No standard at this grade level

6. No standard at this grade level

Process Strands

Strand 5- Problem Solving, Reasoning, and Proof

1. Students will begin to use problem-solving strategies to investigate and understand increasingly complex mathematical content and be able to:

- formulate and solve problems from everyday and mathematical situations.
- explore solving problems using a variety of strategies.
- explore if the solution of a problem is reasonable.
- solve problems using manipulatives and visual representations.
- exhibit confidence in their ability to solve problems independently and in groups.

2. Students will begin to use mathematical reasoning and proof and be able to:

- use models, known facts, properties, and relationships to explain their thinking.
- justify solution processes and answers (e.g. "I know this is a square because….")
- identify the missing information needed to find a solution to a given story problem.

Stand 6- Communication, Connections, and Representations

1. Students will begin to communicate their understanding of mathematics and be able to:

- demonstrate mathematical communication through listening, discussing and responding, individually and in groups.
- recognize and use mathematical language and symbols.
- discuss conclusions and strategies in problem-solving situations.
- draw pictures and use objects to illustrate mathematical concepts.

2. Students will begin to create and use representations to communicate mathematical ideas and to solve problems and be able to:

- create and use age level appropriate representations to organize, record, compare and communicate mathematical ideas (e.g. students should recognize the relationship among seven counters, seven tally marks, and the symbol 7).
- realize that any representation is subject to multiple interpretations (e.g. drawing and graphs can be read in a different way).

3. Students will begin to recognize, explore, and develop mathematical connections and be able to:
- recognize and use mathematics in their daily lives.
- identify mathematical situations occurring in literature for children.
- identify examples of geometry in nature, art, and architecture.

Grade 1

Content Strands

Strand 1- Number and Operations

1. Demonstrates conceptual understanding of rational numbers through the use of models, explanations, or other representations with respect to:

- identifying and using place value concepts of base ten system (for example: group objects into tens and ones, use hundreds chart and place value grid)
- working with fractions as equal shares, using denominators of 2,3,4 and numerators up to the whole
- writing numbers to 100
- recognizing that numbers can be composed and decomposed by applying the concept of place value

2. Demonstrates an understanding of the relative magnitude of numbers from 0 to 100 using models, representations, or number lines by:

- identifying and using simple number patterns when counting by one, two, five, and ten (for example: use number lines, hundred charts and calendar)
- recognizing numbers and exploring quantities 0-100 by connecting concrete materials and abstract representations
- understanding one to one correspondence, counting and conserving values
- being able to compare whole numbers, greater than, less than, equal, one more, and one less utilizing either the number line or the hundred chart

3. Demonstrate conceptual understanding of mathematical operations by:

- developing an understanding of addition and subtraction using concrete materials
- developing, and constructing addition and subtraction equations (number sentences)
- constructing and applying a variety of algorithms
- recognizing and interpreting language and symbols for addition, subtraction and equality

- developing an understanding of the relationship between addition and subtraction
- developing strategies to check reasonableness of solutions in addition and subtraction problems
- solving addition and subtraction problems of whole numbers to 10
- beginning to solve addition of multiple one digit whole numbers
- beginning to recognize the reasonableness of computed sums and differences

4. No standard at this grade level.

5. Demonstrates understanding of monetary value by:

- naming, recognizing and knowing the value of coins (penny, nickel, dime, quarter)
- counting like coins to $1.00

6. Mentally adds and subtracts whole numbers by:

- being able to name the number that is one or two more or less than the original number
- becoming fluent with addition and subtraction facts to 10.

7. Makes estimates by:

- estimating the number of objects in a set up to 30
- revising estimates as objects are counted

8. Applies properties of numbers (odd and even) and field properties (communutative for addition e.g. 2+3=3+2; identity for addition e.g. adding 0 to a number will not change a number) to solve problems and to simplify computations involving whole numbers by:

- identifying odd and even numbers
- making, counting and analyzing sets
- recognizing and demonstrating knowledge of number families to 10

Strand 2--Geometry and Measurement

1. Uses properties, attributes, composition, or decomposition to sort or classify polygons by:

- identifying, sorting and classifying polygons (triangles, squares, rectangles, rhombi, trapezoids and hexagons)
- sorting polygons by a combination of two attributes
- copying and making shapes with drawings and concrete materials

2. No standard at this grade level.

3. Given an example of a three dimensional geometric shape, will be able to:

- identify, sort and classify three-dimensional objects (rectangular prisms, cylinders, spheres, cones and pyramids)

- find an example of three-dimensional objects in the environment

4. **Demonstrates conceptual understanding of congruency by:**

- investigating and observing results of combining and dividing shapes
- constructing geometric patterns and figures using concrete materials
- exploring mirror images and lines of symmetry

5. **No standard at this grade level.**

6. **Demonstrates conceptual understanding of the length/height of a two-dimensional object by:**

- identifying and using non-standard units and tools of measurement
- identifying and using standard units and tools of measurement (inch and centimeter rulers, yard and meter sticks)
- being aware of other standard measurement tools (measuring cups, measuring spoons, scales, calendars, clocks)

7. **Demonstrates conceptual understanding of measurable attributes by:**

- using comparative language to describe and compare attributes of objects (length [longer, shorter], height [taller, shorter], weight [heavier, lighter], temperature [warmer, cooler], and capacity [more, less])
- developing an understanding of the need for standard units of measurement

8. **Determines elapsed and accrued time by :**

- telling time to the hour and half hour, using both digital and analog clocks
- reading the calendar to determine date
- understanding the passage of time as it relates to calendar patterns or the daily schedule (days of the week, months of the year)

9. **Demonstrates an understanding of spatial relationships using location and position by:**

- using positional words (e.g., close by, on the right, underneath, above, beyond) to describe one location in reference to another on a map, in a diagram, and in the environment.

10. **No standard at this grade level.**

Strand 3--Functions and Algebra

1. **Identifies and extends a variety of numeric and non-numeric patterns by:**

- creating a variety of patterns using models, simple rules, concrete materials and numerals (red, yellow, yellow or 5,10,15)
- recognizing, describing, analyzing and extending the above patterns

2. **No standard at this grade level.**

3 **No standard at this grade level.**

4. **Demonstrates conceptual understanding of equality by:**

- using whole numbers to explore the concept of equality and inequality ($<$, $>$, $=$)
- investigating the relationship between addition and subtraction to solve for missing elements in equations ($8+\_\_=12$)
- using a variety of concrete materials, drawings, symbols to represent a number (12 can be a dozen, dice, tallies)

Strand 4--Data, Statistics, and Probability

1. **Create and interpret representations by:**

- collecting, sorting, and organizing data (graphs, charts, tallies, groups)
- using and making graphs and diagrams
- asking and answering questions related to collected data.

2. **Analyzes patterns, trends, or distributions in data in a variety of contexts by:**

- looking for patterns in data using more, less, or equal.

3. **No standard at this grade level.**

4. **No standard at this grade level.**

5. **Demonstrates understanding of probable outcomes by:**

- predicting outcomes
- experimenting with probability (more likely, less likely, or equally likely)

6. **No standard at this grade level.**

Process Strands

Strand 5--Problem Solving, Reasoning, and Proof

1. **Students will use problem-solving strategies to investigate and understand increasingly complex mathematical content and be able to:**

- formulate and solve multi-step problems from everyday and mathematical situations.
- solve problems using a variety of strategies (e.g., working backwards, looking for patterns and relationships; guess and check; making tables, charts, or organized lists; solving a simpler version of a problem, drawing a diagram; or creating a model)
- verify and interpret results with respect to the original problem.
- determine if the solution of a problem is reasonable.

- solve problems using manipulatives, graphs, charts, diagrams, and calculators.
- demonstrate that a problem may be solved in more than one way.
- exhibit confidence in their ability to solve problems independently and in groups.
- display increasing perseverance, and persistence in problem solving.

2. **Students will use mathematical reasoning and proof and be able to:**

- use models, known facts, properties, and relationships to explain their thinking.
- justify solution processes and answers (e.g., "I chose this method to solve the problem because…").
- draw conclusions using inductive reasoning.
- identify the missing information needed to find a solution to a given story problem.
- use patterns and relationships to analyze mathematical situations (e.g., count by fives).

Strand 6--Communication, Connections, and Representations

1. Students will communicate their understanding of mathematics and be able to:

- demonstrate mathematical communication through discussion, reading, writing, listening, and responding, individually and in groups.
- discuss relationships between everyday language and mathematical language and symbols (e.g., words that mean something different in mathematics and in everyday life).
- explain conclusions, thought processes, and strategies in problem-solving situations.
- discuss, illustrate, and write about mathematical concepts and relationships.
- draw pictures and use objects to illustrate mathematical concepts.

2. Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:

- create and use age level appropriate representations to organize, record, and communicate mathematical ideas (e.g., students should recognize the relationship among seven counters, seven tally marks, and the symbol 7).
- select, apply, and translate among mathematical representations to solve problems (e.g., representing fractions with circles, with geoboards, and with pattern blocks).
- link different representations.
- use representations to model and interpret physical, social, and mathematical phenomena.
- use conventional and self-generated (invented) representations and connect them.
- realize that any representation is subject to multiple interpretations (e.g., drawings and graphs can be read in a different way).

3. **Students will recognize, explore, and develop mathematical connections** and be able to:

- link conceptual and procedural knowledge (e.g., they will know that when they "regroup," they are simply changing the representation of the minuend, but not its value).
- recognize and use mathematics in other curriculum areas (e.g., science, social studies).
- recognize and use mathematics in their daily lives (e.g., graphs, tables, or maps).

- identify mathematical situations occurring in literature for children.
- identify examples of geometry in nature, art, and architecture.

Grade 2

Content Strands

Strand 1 - Number and Operations

1. Demonstrates conceptual understanding of rational numbers through the use of models, explanations, or other representations with respect to:

- ✓ understanding whole numbers from 0 to 199 using place value
- ✓ applying concepts of equivalency in composing or decomposing numbers (e.g. 34 = 17 + 17; 34 = 29 + 5)
- ✓ expanding notation (e.g. 141 = 1 hundred + 4 tens + 1 one or 141 = 100 + 40 + 1)
- ✓ investigating numbers to 1,000 (e.g. order, make using models, and write in standard form)
- ✓ demonstrating an understanding of the base ten number system
- ✓ demonstrating understanding of positive fractional numbers (benchmark fractions: a/2, a/3, or a/4, where a is a whole numbers greater than 0 and less than or equal to the denominator) as a part to whole relationship
- ✓ understanding that the denominator is equal to the number of parts in the whole
- ✓ recognizing fractions as equal shares

2. Demonstrates an understanding of the relative magnitude of numbers from 0 to 199 using models, representations, or number lines by:

- ✓ reading, writing and ordering whole numbers (e.g. name whole number immediately before or after a number)
- ✓ comparing whole numbers to each other or to benchmark whole numbers (10, 25, 50, 75, 100, 125, 150, or 175);
- ✓ demonstrating an understanding of the relation of inequality when comparing whole numbers by using "1 more", "1 less", "10 more", "10 less", "100 more", or "100 less"
- ✓ comparing any two-digit or three-digit numbers to determine which one is larger or smaller
- ✓ connecting number words and numerals to the quantities they represent using concrete materials, models, number lines, or explanations.
- ✓ understanding one-to-one correspondence, counting (using ordinal and cardinal numbers), and conserving values (e.g. 2+3=1+4

3. Demonstrates conceptual understanding of mathematical operations by:

- ✓ understanding mathematical operations involving addition and subtraction of whole numbers by solving problems involving joining actions, separating actions, part-part whole relationships, and comparison situations
- ✓ solving addition and subtraction facts to 20
- ✓ understanding addition of multiple one-digit whole numbers
- ✓ developing and using a variety of strategies to add and subtract numbers
- ✓ constructing addition and subtraction algorithms
- ✓ recognizing and interpreting symbols for addition, subtraction, equal to, greater than, and less than
- ✓ investigating regrouping to the tens place for addition and subtraction

4. No standard at this grade level.

5. Demonstrate understanding of monetary value by:

- ✓ recognizing and naming coins (penny, nickel, dime, and quarter) and one dollar bill
- ✓ identifying values and relationships among coins
- ✓ adding coins together to a value up to $1.99 and represent the result in dollar notation;
- ✓ making change from $1.00 or less
- ✓ recognizing equivalent coin representations of the same value (values up to $1.99).

6. Mentally add and subtract whole number facts by:

- adding and subtracting whole numbers through 20 (addends whose sum is at most 20 and related subtractions facts)
- naming the number that is 10 more or 10 less than the original number, and mentally adding and subtracting two-digit multiples of ten (e.g., 60 + 80, 90 – 30).
- becoming fluent with addition and subtraction facts to 14
- taking advantage of opportunities to integrate/use numeration skills throughout the day investigating the properties of operations to solve related problems (for example, number families, doubles plus or minus one)

    (IMPORTANT: The intent of this proficiency is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.)

7. Make estimates of the number of objects in a set (up to 50) by selecting an appropriate method of estimation.

    (IMPORTANT: The intent of this proficiency is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.)

8. Applies properties of numbers (odd and even) and field properties [commutative for addition, e.g. 2+3 = 3+2; identity for addition e.g. adding zero to a number will not change the number; and associative for addition, e.g. (1+2) +3 = 1+ (2+3)] to solve problems and to simplify computations involving whole numbers by:

- understanding and counting by odd and even numbers

Strand 2 - Geometry and Measurement

1. Uses properties, attributes, composition (putting shapes together to form various shapes), or decomposition (breaking down a shape into smaller shapes) to sort or classify polygons or objects by a combination of two or more non-measurable or measurable attributes by:

- ✓ investigating angles, parallel lines, and intersecting lines
- ✓ investigating geometric vocabulary (e.g., polygon, parallelogram, faces, base, quadrilateral, quadrangle,)
- ✓ demonstrating an understanding of the following geometric vocabulary: vertical, horizontal, diagonal
- ✓ demonstrating an understanding of the following 2-D shapes by being able to copy and make shapes using concrete materials or drawing: circle, square, rectangle, triangle, rhombus, trapezoid, hexagon and octagon
- ✓ demonstrating an understanding of the following 3-D shapes by being able to name and identify attributes: cube, sphere, cone, rectangular prism
- ✓ investigating triangular based and square based pyramids

2. No standard at this grade level.

3. No standard at this grade level.

4. Demonstrate conceptual understanding of congruency by:

- composing and decomposing two-dimensional objects using models or explanations (e.g. using triangular pattern blocks to construct a figure that is the same size and same shape as the hexagonal pattern block)
- using line symmetry and mirrors to demonstrate congruent parts within a shape.
- constructing and extending geometric patterns and figures using concrete materials

5. No standard at this grade level.

6. Demonstrates conceptual understanding of perimeter and area by:

- ✓ using models or manipulatives to surround and cover polygons.

7. Measure and use units of measures appropriately and consistently, and make conversions within systems when solving problems across the content strands by:

- ✓ demonstrating an understanding of the need for standard units of measurement.
- ✓ investigating the use of the appropriate tool for the task (thermometer, ruler, yardstick, meter stick, balance scales, regular scale)
- ✓ using standard measurement units and tools, including metric [Unit accuracy: inch (to whole inch), foot (to whole inch); centimeter (to whole centimeter); meter (to whole centimeter)] [Equivalencies: 12 inches in 1 foot; 100 centimeters in 1 meter]
- ✓ telling time to the quarter hour on analog/digital clock [unit accuracy: hour (to 15 minute interval); [Equivalencies: 60 minutes in 1 hour]
- ✓ reading calendar to determine day, month, and year, and beginning to associate important events with each month

- ✓ investigating the concept of the passage of time including seasons, months, days, hours and minutes
- ✓ reading the temperature to one degree Fahrenheit

8. No standard at this grade level.

9. Demonstrates an understanding of spatial relationships using location and position by:
- using positional language in two- and three- dimensional situations to describe and interpret relative positions (e.g., above the surface of the desk, below the triangle on the paper, between the desk and the door)
- creating and interpret simple maps and name locations on simple coordinate grids

11. No standard at this grade level.

Strand 3 - Functions and Algebra

1. Identify and extend a variety of linear and non-numeric patterns. (*A linear pattern is a sequence of numbers with a constant rate of change. e.g. 2,4,6,8,10*) by:

- ✓ using models, tables, concrete materials or sequences to extend patterns to the next element, or finding a missing element (e.g., 2, 4, 6, _____, 10)

2. No standard at this grade level.

3. No standard at this grade level.

4. Demonstrate conceptual understanding of equality by:

- ✓ finding the value that will make an open sentence true (e.g., 2+ =7) limited to one operation and limited to use of addition or subtraction

Strand 4 - Data, Statistics, and Probability

- ✓ **1. Interpret a given representation (pictographs with one-to-one correspondence, line plots, tally charts, or tables) to answer questions related to the data, or to analyze the data to formulate conclusions.**

- ✓ **2. Analyze patterns, trends, or distributions in data in a variety of contexts by determining or using more, less, or equal.**

3. No standard at this grade level.

- ✓ **4. Use counting techniques to solve problems involving combinations using a variety of strategies (e.g. student diagrams, organized lists, tables, tree diagrams, or others); (e.g., How many ways can you make 50 cents using nickels, dimes, and quarters?)**

5. For a probability event in which the sample space may or may not contain equally likely outcomes, use experiments to describe the likelihood or chance of an event using "more likely," "less likely," "equally likely," certain or impossible.

6. In response to a teacher or student generated question or hypothesis, groups decide the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical) from a variety of sources, (e.g. real life, literature) necessary to answer the question; collect, organize and appropriately display the data; analyze the data to draw conclusions about the question or hypothesis being tested, and when appropriate makes predictions by:

- making graphs and diagrams (Venn diagrams, bar graphs, pie graphs)
- representing the same data in a variety of ways
- identifying and labeling vertical and horizontal axes on graphs, and giving the graph a title

Process Strands

Strand 5 - Problem Solving, Reasoning, and Proof

1. Students will use problem-solving strategies to investigate and understand increasingly complex mathematical content and be able to:

- formulate and solve multi-step problems from everyday and mathematical situations
- solve problems using a variety of strategies (e.g., working backwards, looking for patterns and relationships; guess and check; making tables, charts, or organized lists; solving a simpler version of a problem, drawing a diagram; or creating a model)
- verify and interpret results with respect to the original problem
- develop strategies to determine if the solution of a problem is reasonable
- use a calculator to check solutions
- solve problems using manipulatives, graphs, charts, diagrams, and calculators
- demonstrate that a problem may be solved in more than one way
- exhibit confidence in their ability to solve problems independently and in groups
- display increasing perseverance, and persistence in problem solving

2. Students will use mathematical reasoning and proof and be able to:

- use models, known facts, properties, and relationships to explain their thinking
- justify solution processes and answers (e.g., "I chose this method to solve the problem because…")
- draw conclusions using inductive reasoning (create broader generalization from specific problems; e.g. solving problems with even numbers added together and making a generalization that an even number plus another even number will always add up to an even number)
- identify the missing information needed to find a solution to a given story problem
- use patterns and relationships to analyze mathematical situations (e.g., count by fives)

Strand 6 - Communication, Connections, and Representations

1. Students will communicate their understanding of mathematics and be able to:

- demonstrate mathematical communication through discussion, reading, writing, listening, and responding individually and in groups
- discuss relationship between everyday language and mathematical language and symbols (e.g. words that mean something different in mathematics and in everyday life--e.g. nothing/zero, sum/some, point)
- explain conclusions, thought processes, and strategies in problem-solving situations
- discuss, illustrate, and write about mathematical concepts and relationships.
- draw pictures and use objects to illustrate mathematical concepts.

2. Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:

- create and use age level appropriate representations to organize, record, and communicate mathematical ideas (e.g., students should recognize the relationship among seven counters, seven tally marks, and the symbol 7)
- select, apply, and translate among mathematical representations to solve problems (e.g., representing fractions with circles, with geoboards, and with pattern blocks)
- understand how these different representations show the same concept
- use representations to model and interpret physical, social, and mathematical phenomena (e.g. make a chart to show the seating arrangement in a class)
- use conventional and self-generated (invented) representations and connect them
- realize that any representation is subject to multiple interpretations (e.g., drawings and graphs can be read in a different way)

3. Students will recognize, explore, and develop mathematical connections and be able to:

- link conceptual and procedural knowledge (e.g., they will know that when they "regroup", they are simply changing the representation of the minuend, but not its value)
- recognize and use mathematics in their daily lives (e.g., graphs, tables, or maps)
- identify mathematical situations occurring in literature for children
- listen to and read books related to mathematics
- identify examples of geometry in nature, art, and architecture

GRADE 3

Content Strands

Strand 1-Number and Operations

1. Demonstrates conceptual understanding of rational numbers through the use of models, explanations, or other representations with respect to:

- ✓ whole numbers from 0 -10,000 through equivalency, composition, decomposition or place value;
- ✓ positive fractional numbers (benchmark fractions: a/2, a/3, a/4, a/5, a/6, a/7, a/8,
- ✓ where a is a whole number greater than 0 and less than or equal to the denominator) as a part to whole relationship in area and set models where the number of parts in the whole is equal to the denominator;
- ✓ decimals (within a context of money) as part of 100.

2. Demonstrates an understanding of the relative magnitude of numbers from 0 to 10,000 using models, representations, or number lines by:

- ✓ ordering whole numbers
- ✓ comparing whole numbers to benchmark whole numbers (100, 250, 500, 750)
- ✓ comparing whole numbers to each other (equal to, greater than, and less than)
- ✓ comparing or identifying equivalent positive fractional numbers (a/2, a/3, a/4, where a is a whole number greater than 0 and less than or equal to the denominator).

.

3. Demonstrates conceptual understanding of mathematical operations using models, number lines, or explanations by:

- ✓ describing or illustrating the inverse relationship between addition and subtraction of whole numbers

✓ describing or illustrating the relationship between repeated addition and multiplication.

4. Accurately solves problems involving:

✓ a variety of strategies to add and subtract numbers
✓ addition and subtraction with regrouping with three digit numbers
✓ the concept of multiplication and division
✓ addition or subtraction of decimals (in the context of money)
✓ making change with appropriate coins and bills
✓ recognizing equivalent coin representation of the same value (e.g. various coin combinations to make 50¢).

5. No standard at this grade level.

6. Mentally adds and subtracts whole number facts:

- becomes fluent with addition and subtraction facts through 20
- becomes fluent with multiplication and division facts through 5x5 and 25÷5
- investigates the properties of operations to solve related problems (e.g. number families, double plus or minus one)
- through 20 (addends whose sum is at most 20 and related subtraction facts)
- adds two-digit and one-digit whole numbers
- adds combinations of two-digit and three-digit whole numbers that are multiples of ten (e.g. 60+50, 300+400, 320+90)
- subtracts a one-digit whole number from a two-digit whole number (e.g. 37-5)
- subtracts two-digit whole numbers that are multiples of ten and three-digit whole numbers that are multiples of one hundred (e.g. 50-20, 500-200)
  (The intent of this proficiency is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.)

7. Makes estimation in a given situation by:

- identifying when estimation is appropriate
- selecting the appropriate method of estimation
- and evaluating the reasonableness of solutions appropriate to grade level proficiencies across content strands.
  (The intent of this proficiency is to embed estimation throughout the instructional program, not to teach it as a separate unit.)

8. Applies properties of numbers to solve problems and to simplify computations involving whole numbers:

- odd and even
- multiplicative property of zero for single-digit whole numbers (6x0=0)
- field properties (communicative for addition, associative for addition, identity for multiplication, and commutative for multiplication for single-digit whole numbers (e.g. 3x4=4x3).
- Know for quick recall:

- addition and subtraction fact through 20
- multiplication facts to 5x5
- division facts to 25/5.
- Demonstrate knowledge of number families to eighteen.
- Demonstrate strategies for solving facts to 10x10.
- Use properties of operations to solve related problems (for example: communicative property).

Strand 2 - Geometry and Measurement

1. Uses properties or attributes of angles (number of angles) or sides (number of sides or length of sides or vertices) or composition or decomposition of shapes to identify, describe, or distinguish among triangles, squares, rectangles, rhombi, trapezoids, hexagons or circles.

- ✓ Investigate geometric vocabulary including polygon, quadrangles, quadrilaterals
- ✓ Name, describe, and classify angles (for example: right, acute, obtuse, and straight).

2. No standard at this grade level.

3. No standard at this grade level.

4. Demonstrates conceptual understanding of congruency by:

- matching congruent figures using reflections, translations, and rotations (flips, slides and turns) (e.g. recognizing when pentominoes are reflections   translations, and rotations of each other)
- composing and decomposing two- and three- dimensional objects using models or explanations (e.g. Given a cube, students use blocks to construct a congruent cube)
- using line symmetry to demonstrate congruent parts within a shape.

5. Demonstrates conceptual understanding of similarity by identifying similar shapes.

6. Demonstrates conceptual understanding of:

- ✓ perimeter of polygons
- ✓ area of rectangles on grids using a variety of models and manipulatives
- ✓ the need to express all measures using appropriate units
- ✓ use the terms point, line, and line segment
- ✓ parallel, perpendicular, and intersecting lines.

✓ **7. Measures and uses units of measures appropriately and consistently and makes conversions within systems when solving problem across content strands.**

Measure Benchmarks for Grade 3:

Length:
Unit (accuracy): Inch (to ½ inch); Foot (to whole inch); Centimeter (to whole centimeter); Meter (to whole centimeter)
Equivalencies: 12 inches in 1 foot; 100 centimeters in 1 meter

Time:

Unit (accuracy): Hour (to 5 minute interval); Day; Year
Equivalencies: 24 hours in 1 day; 7 days in 1 week; 365 days in 1 year

Temperature:
Unit (accuracy): Cº and Fº (to 1 degree)

Capacity:
Units (accuracy): Quart (to whole quart)

Mass:
Unit (accuracy): Kilogram (to whole kilogram); Gram (to whole gram)

Weight:
Unit (accuracy): Pound (to whole pound)

8. No standard at this grade level.

9. Demonstrates understanding of spatial relationships using locations and position by:

- interpreting and giving directions from one location to another (e.g. classroom to the gym, from school to home) using positional words
- between locations on a map or coordinate grid (first quadrant) using positional words or compass directions.

10. Demonstrates conceptual understanding of spatial reasoning and visualization by:

- copying, comparing, and drawing models of triangles, squares, rectangles, rhombi, trapezoids, hexagons, and circles
- name and identify attributes of three dimensional shapes including rectangular prism, cylinder, sphere, cone, pyramid, cube
- use the terms base, face, edge and vertex.

Strand 3-Functions and Algebra

1. Identifies and extends to a variety of linear and non-linear patterns (A linear pattern is a sequence of numbers with a constant rate of change, e.g. 2, 4, 6, 8, 10. A non-linear pattern is a sequence of numbers with varying rates of change, e.g. $x^2 = y$) represented in: models, tables, or sequences by extending the pattern to the next one, two, or three elements or finding missing elements.

2. No standard at this grade level.

3. No standard at this grade level.

4. Demonstrates conceptual understanding of equality by:

- ✓ showing equivalence between two expressions using models or different representations of the expressions
- ✓ finding the value that will make an open sentence true (e.g. 2 + ? = 7). (Limited to one operation and limited to using addition, subtraction or multiplication).

Strand 4- Data, Statistics, Probability

✓ **1. Interprets a given representation (line plot, tally chart, tables, or bar graphs) to answer questions related to data, analyze data to formulate conclusions, or make predictions.**

✓ **2. Analyze patterns, trends, or distributions in data in a variety of contexts by determining or using: most frequent (mode), least frequent, (maximum) largest, smallest (minimum).**

3. Organizes and displays data using: tables, tally charts, and bar graphs, to answer questions related to data, to analyze the data to formulate conclusions, to make predictions, or to solve problems.

   ✓ Given a set of data, identifies the best representation to display the data.
.
4. Uses counting techniques to solve problems involving combinations and simple permutations using a variety of strategies (e.g. student diagrams, organized lists, tables, or other).

✓   **5. For a probability event in which the sample space may or may not contain equally likely outcomes, predicts the likelihood of an event using "more likely", "less likely", "equally likely", "certain" or "impossible" and tests the prediction through experiments; and determines if a game is fair.**

6. In response to a teacher or student generated question or hypothesis, groups decide the most effective method (e.g. survey, observation, experimentation) to collect the data (numerical or categorical) necessary to answer the question; collects, organizes and appropriately displays the data; analyzes the data to draw conclusions about the question or hypothesis being tested, and when appropriate makes predictions.

Process Strands

Strand 5 - Problem Solving, Reasoning, and Proof

1. Students will use problem-solving strategies to investigate and understand increasingly complex mathematical content and be able to:

- determine and explain the reasonableness of solutions to real-world problems
- generalize solutions and apply strategies to new problem situations.
- add to the repertoire of problem-solving strategies (e. g. looking for similar problems) and use those strategies in more sophisticated ways*

- solve problems with multiple solutions, recognize when a problem has no solution, and recognize problems where more information is needed
- translate results of a computation into solutions that fit the real-world
- problem (e. g. when a computation shows that one needs 3.2 gallons of paint to paint a room, how much paint do you buy?).

2. Students will use reasoning and proof and be able to:

- draw conclusions and solve problems using deductive reasoning and reasoning by analogy
- make and defend conjectures and generalizations
- use models, known facts, properties and relationships to explain thinking and justify answers and processes
- recognize the power of reasoning as a part of mathematics.

3. Students will understand the following classifications of word problems:

Join: (13 - 5 = __,   5 + __ = 13, __ + 5 = 13)
Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have altogether?   (13 - 5 = __)

Separate: (13 - 5 = __, 13 - __ = 5, __ - 5 = 8)
Connie has some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to start with? (__ - 5 = 8)

Part-part-whole: (5 + 8 = __, 13 – 5 = __)
Connie has 13 marbles. All the marbles are either blue or red. Connie has 5 red marbles. How many blue marbles does Connie have?

Compare:
Juan has 5 marbles. Connie has 8 more marbles than Juan. How many marbles does Connie have? (5 + 8 = ___)

Strand 6 - Communications, Connections, and Representations

1. Students will communicate their understanding of mathematics and be able to:
- use the language of mathematics appropriate to the mathematical strand
- discuss and explain mathematical thinking
- discuss and solve a variety of real life mathematics problems
- use a variety of strategies to solve problems
- apply numeration skills throughout the day
- ask clarifying and extending questions related to mathematics they have heard or read about
- demonstrate an understanding of mathematical concepts and relationships through a variety of methods (e.g. writing, graphing, charts, diagrams, number sentences or symbols).
- use a variety of technologies (e.g. computers, calculators, videos) to represent and communicate mathematical ideas.

2. Students will create and use representations to communicate mathematical ideas to solve problems and be able to:
- use physical models and diagrams to represent important mathematical ideas (e. g. multiplication)
- use appropriate representations to solve problems or to portray, clarify, or extend a mathematical idea.

Grade 4

Content Strands

Strand 1 - Number and Operations

1. Demonstrates conceptual understanding of rational numbers through the use of models, explanations, or other representations with respect to:

- ✓ knowing whole numbers from 0 – 999,999 through equivalence, composition, decomposition (i.e., 24 = 13 + 11) and place value
- ✓ understanding and identifying proper fractions (i.e., ½, 7/8, 8/10) and equivalent fractions
- ✓ understanding decimals as hundredths related to money or tenths related to metric measurements (2.3 cm)
- ✓ understanding prime and composite numbers.

2. Demonstrates an understanding of the relative magnitude of numbers from 0 to 999,999 using models, representations, or number lines by:

- ✓ ordering and comparing whole numbers, decimals, and fractions.

3. Demonstrates conceptual understanding of mathematical operations using models, number lines, or explanations by:

- ✓ understanding the inverse relationship between multiplication and division
- ✓ adding and subtracting fractional amounts with like denominators
- ✓ understanding the relationship between division and repeated subtraction as well as multiplication and repeated addition.

4. Accurately solves problems involving :

- ✓ the properties of factors and multiples
- ✓ 2-and 3-digit x 1-digit multiplication

- ✓ 2-digit x 2-digit multiplication
- ✓ division using 1-digit divisors.
- ✓ subtraction involving regrouping with 4-digit numbers
- ✓ addition and subtraction of decimal amounts
- ✓ multiple operations
- ✓ choosing the most efficient method to solve problems.

5. **No standard at this grade level.**

6. **Mentally adds and subtracts whole number facts:**

- adds and subtracts whole numbers through 20
- adds problems such as 67 + 24, 320 + 430, and 1300 + 1400
- subtracts problems such as 67- 9, 50 -20, 230 – 80, 520 – 200
- is fluent with multiplication and division facts through 10 x 10 and 100 / 10.

7. **Makes estimation in a given situation by:**

- estimating answers and evaluating reasonableness of solutions.

8. **Applies properties of numbers to solve problems and to simplify computation involving whole numbers:**

- understands and applies properties of numbers (odd, even, multiplicative property of zero, remainders)
- understands and applies commutative property (2 x 4 = 4 x 2 , 3 + 4 = 4 + 3), identity property (1 x 12 = 12) and associative property 2 + (3 + 4) = (2 + 3) + 4
- understands order of operations, including use of parentheses.

Strand 2 -Geometry and Measurement

1. **Uses properties or attributes of angles or sides to identify, describe, or distinguish among:**

- ✓ number of angles or sides, length of sides, parallelism, or perpendicularity among triangles, squares, rectangles, rhombi, trapezoids, hexagons, or octagons
- ✓ acute, obtuse, and right angles
- ✓ lines, segments, points, rays, parallel and perpendicular lines.

2. **No standard at this grade level.**

3. **Uses properties or attributes to identify, compare, or describe 3-dimentional shapes:**

- ✓ uses properties such as shape of bases or number of faces to identify, compare, describe, and build 3-dimentional shapes such as rectangular prisms, triangular prisms, cylinders, and spheres.

4. **Demonstrates conceptual understanding of congruency:**

✓ matches congruent figures using reflections, translations, and rotations (i.e., flips, slides, and turns).

5. Demonstrates conceptual understanding of similarity by:

✓ applying scales on maps, or applying characteristics of similar figures (same shape but not necessarily same size).

6. Demonstrates conceptual understanding of:

✓ perimeter of polygons
✓ area of rectangles, polygons, or irregular shapes on grids using a variety of models and manipulatives
✓ the need to express all measures using appropriate units (i.e., area is expressed in square units).

7. Measures and uses units of measures appropriately and consistently and makes conversions within systems when solving problems:

✓ uses appropriate unit of measurement (i.e., inches or cm to measure something small, yards or meters to measure larger items or spaces) to accurately determine size or arrive at solutions
✓ understands the following measurement benchmarks:

Length
Unit (accuracy): inch to ¼ inch, foot, centimeter to .5 cm, meter to .5 cm, yard, mile, kilometer

Equivalencies: 12 inches in 1 foot, 100 cm in 1 meter, 3 feet in 1 yard, 36 inches in 1 yard

Time
Unit (accuracy): hour to 5 minutes intervals, day, year

Equivalencies: 24 hours in 1 day, 7 days in 1 week, 35 days in 1 year, 60 seconds in 1 minute, 60 minutes in 1 hour

Temperature
Unit (accuracy): C and F to 1 degree

Capacity
Unit (accuracy): Quart (to whole quart)

Mass
Unit (accuracy): Kilogram (to whole Kg), gram (to whole g)

Weight
Unit (accuracy): Pound (to whole pound)

8. **No standard at this grade level.**

9. **Demonstrates understanding of spatial relationships using location and position by:**

    - interpreting and giving directions from one location to another (e.g. classroom to the gym, from school to home) using positional words.
    - finding locations on a map or coordinate grid (first quadrant) using positional words or compass directions.

10. **Demonstrates conceptual understanding of spatial reasoning and visualizations:**

    - copies and draws models of two dimensional shapes and builds models of two and three dimensional shapes.

Strand 3 - Functions and Algebra

1. **Demonstrates the ability to identify and extend a variety of linear and non-linear patterns (A linear pattern is a sequence of numbers with a constant rate of change, e.g. 2, 4, 6, 8, 10. A non-linear pattern is a sequence of numbers with varying rates of change, e.g. $x^2 = y$.) by:**

    - ✓ representing them in models, tables, or sequences
    - ✓ writing a rule in words or symbols to find the next case.

2. **Demonstrates a conceptual understanding of linear relationships as a constant rate of change by:**

    - describing and comparing situations that represent constant rates of change**.**

3. **Demonstrates a conceptual understanding of algebraic expressions by:**

    - ✓ using letters or symbols to represent unknown quantities
    - ✓ writing simple linear expressions involving any one of the 4 operations
    - ✓ evaluating expressions using whole numbers.

4. **Demonstrates conceptual understanding of equality by:**

    - ✓ showing equivalence between two expressions using models or different representations of the expressions
    - ✓ simplifying numerical expressions involving (e.g. [14-(2x5)]
    - ✓ solving one step linear equations (i.e., solves for b if 5b = 30).

Strand 4 –Data, Statistics, and Probability

✓ **1. Interprets a given representation** (line plot, tally chart, tables, pictographs, circle graphs, or bar graphs) to answer questions related to data, analyze data to formulate conclusions, make predictions or solve problems.

✓ **2. Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using median, mode and range.**

3. Organizes and displays data using line plots, bar graphs, pictographs, and circle graphs; answers questions related to the data; analyzes data to formulate conclusions, make predictions or solve problems.

4. Uses counting techniques to solve problems:

✓ in context involving combinations or simple permutations (i.e., If you have 5 colors of shirts, and 5 colors of shorts, how many different outfits can be made?) using a variety of strategies (such as lists, tables, tree graphs, or others).

5. For a probability event for which the sample space may or may not contain equally likely outcomes, predicts and determines:

✓ the likelihood of an event as a part to whole relationship (2 out of 5), examines theoretical probability, tests the prediction through experiments, and determines fairness of a game or activity.

6. In response to a teacher or student generated question or hypothesis, groups will:

- determine the best method to collect data
- collect, organize, and display data
- analyze data and draw conclusions
- make predictions and ask new questions
- make connections to real world situations.

Process Strands

Strand 5 - Problem Solving, Reasoning, and Proof

1. Students will use a variety of problem-solving strategies to investigate and understand increasingly complex mathematical content and be able to:

- determine reasonableness of solutions
- generalize solutions and apply strategies to new problems
- solve problems with multiple solutions, recognize when a problem has no solutions, and recognize problems where more information is needed
- translate results of computation into solutions that fit real-world problems (i.e., "If there are 27 people going on a trip and 5 people can fit into a van, how many vans are needed? Answer = 6, not 5 R2).

2. Students will use reasoning and proof and be able to:

- draw conclusions and solve problems using deductive reasoning and reasoning by analogy
- make and defend conjectures and generalizations
- use models, known facts, properties and relationships to explain thinking and justify answers and processes
- recognize the power of reasoning as a part of mathematics.

3. Students will understand the following classifications of word problems:

<u>Join:</u> $(13 - 5 = \_\_,\ \ 5 + \_\_ = 13,\ \_\_ + 5 = 13)$

Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have altogether? $(13 - 5 = \_\_)$

<u>Separate:</u> $(13 - 5 = \_\_,\ 13 - \_\_ = 5,\ \_\_ - 5 = 8)$

Connie has some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to start with? $(\_\_ - 5 = 8)$
<u>Part-part-whole:</u> $(5 + 8 = \_\_,\ 13 - 5 = \_\_)$

Connie has 13 marbles. All the marbles are either blue or red. Connie has 5 red marbles. How many blue marbles does Connie have?

<u>Compare:</u>

Juan has 5 marbles. Connie has 8 more marbles than Juan. How many marbles does Connie have? $(5 + 8 = \_\_)$

Strand 6 -Communication, Connections, and Representations

1. Students will communicate their understanding of mathematics and be able to:

- discuss mathematical ideas and write arguments
- understand, explain, analyze, and evaluate arguments and conclusions
- ask questions related to mathematics they have heard or read about
- demonstrate an understanding of concepts and relationships through writing, graphing, charts, diagrams, number sentences, and symbols
- use a variety of technologies (including calculators) to represent and communicate ideas.

2. Students will create and use representations to communicate mathematical ideas and to solve problems:

- use models and diagrams to represent ideas
- use representations to solve problems or to portray, clarify, or extend an idea

- recognize equivalent representations of concepts and procedures and translate among them (i.e., learning fractions via fraction bars, blocks/manipulatives, and number representations like 3/5, and being able to translate one to the other).

3. **Students will recognize, explore, and develop mathematical connections and be able to:**

- see mathematics as a whole
- recognize relationships among different topics
- use math in other curriculum areas and in their daily lives
- link concepts and procedures.

Grade 5

Content Strands

Strand 1 - Number and Operations

1. **Demonstrates conceptual understanding of rational numbers with respect to:**

- whole numbers from 0 to 9,999,999 through equivalency, composition, decomposition, or place value using models, explanations, or other representations;
- positive fractional numbers (proper, mixed number, and improper) (halves, fourths, eighths, thirds, sixths, twelfths, fifths)
- powers of ten (10, 100, 1000),
- decimals (to thousandths),
- benchmark percents (10%, 25%, 50%, 75% or 100%) as a part to whole relationship in area, set, or linear models using models, explanations, or other representations.

2. **Demonstrates understanding of the relative magnitude of numbers by ordering, comparing, or identifying:**

- equivalent positive fractional numbers
- decimals
- benchmark percents within number formats (fractions to fractions, decimals to decimals, or percents to percents)
- integers in context using models or number lines

3. **Demonstrates conceptual understanding of mathematical operations by:**

- adding and subtracting decimals and positive proper fractions with unlike denominators.
- describing or illustrating the meaning of a remainder with respect to division of whole numbers using models, explanations, or solving problems.

4. **Accurately solves problems involving:**

- multiple operations on whole numbers
- the use of the properties of factors
- multiples
- prime and   composite numbers
- addition or subtraction of fractions (proper)
- decimals to the hundredths place.
- Division of whole numbers by up to a two-digit divisor.
  (IMPORTANT: Proficiency applies the conventions of order of operations with and without parentheses.)

5. **No standard at this grade.**

6. **Mentally calculates:**

- change back from $1.00, $5.00, and $10.00;
- calculates multiplication and related division facts to a product of 144;
- multiplies a two-digit whole number by a one-digit whole number (e.g., 45×5),
- two-digit whole numbers that are multiples of ten (e.g., 50×60),
- a three-digit whole number that is a multiple of 100 by a two- or three-digit number which is a multiple of 10 or 100, respectively (e.g., 400×50, 400×600);
- divides 3- and 4-digit multiples of powers of ten by their compatible factors (e.g., $360 \div 6$; $360 \div 60$; $3600 \div 6$; $3600 \div 60$; $3600 \div 600$; $360 \div 12$; $360 \div 120$; $3600 \div 12$; $3600 \div 120$; $3600 \div 1200$).

7. **Makes estimates in a given situation by:**

- identifying when estimation is appropriate
- selecting the appropriate method of estimation
- determining the level of accuracy needed given the situation,
- analyzing the effect of the estimation method on the accuracy of results,
- evaluating the reasonableness of solutions

8. **Applies properties to solve problems and to simplify computaions with:**

- numbers (odd, even, and divisibility)
- field properties (commutative, associative, identity, and distributive)

Strand 2   -   Geometry and Measurement

1. **Uses properties or attributes to identify, describe, classify, or distinguish among:**

    - **angles** (right, acute, or obtuse)
    - **sides** (number of congruent sides, parallelism, or perpendicularity)
    - triangles (right, acute, obtuse, equiangular, or equilateral)
    - quadrilaterals (rectangles, squares, rhombi, trapezoids, or parallelograms).

2. **No standard at this grade.**

3. **Uses properties or attributes to identify, compare, or describe three-dimensional shapes:**

    - shape of bases, number of lateral faces, or number of bases
    - rectangular prisms, triangular prisms, cylinders, spheres, pyramids, or cones.

4. **No standard at this grade.**

5. **Demonstrates conceptual understanding of similarity by:**

    - describing the proportional effect on the linear dimensions of triangles and rectangles when scaling up or down while preserving angle measures,
    - solving related problems (including applying scales on maps).

6. **Demonstrates conceptual understanding of:**

    - perimeter of polygons
    - area of rectangles or right triangles through models, manipulatives, or formulas
    - area of polygons or irregular figures on grids,
    - volume of rectangular prisms (cubes) using a variety of models, manipulatives, or formulas.
    - Expresses all measures using appropriate units.

7. **Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems across the content strands.**

Measures:	Grade 5
Length	**Units (accuracy):** Inch (to 1/8 inch); Foot; Centimeter (to 0.5 centimeter); Meter (to 0.5 centimeter); Yard; Mile (use in scale questions); Kilometer (use in scale questions) **Equivalencies:** 12 inches in 1 foot; 100 centimeters in 1 meter; 3 feet in 1 yard; 36 inches in 1 yard; 10 millimeters in 1 centimeter
Time	**Unit (accuracy):** Hour (to 1 minute); Day; Year **Equivalencies:** 24 hours in 1 day; 7 days in 1 week; 365 days in 1 year; 60 seconds in 1 minute; 60 minutes in 1 hour
Temperature	**Unit (accuracy):** Cº and Fº (to 1 degree)
Capacity	**Unit (accuracy):** Quart (to 1 ounce); Gallon; Pint **Equivalencies:** 32 ounces in 1 quart; 4 quarts in 1 gallon; 2 pints in 1 quart

Mass	**Unit (accuracy):** Kilogram; Gram (to whole gram)
Weight	**Unit (accuracy):** Pound (to 1 ounce) **Equivalencies:** 16 ounces in 1 pound
Angles and Rotation	**Unit (accuracy):** Degree (to 2 degrees) **Equivalencies:** 360^0 in 1 circle; 90^0 in 1 right angle

8. No standard at this grade.

9. Demonstrates understanding of spatial relationships using location and position by:

- interpreting and giving directions between locations on a map or coordinate grid (all four quadrants);
- plotting points in four quadrants in context (e.g., games, mapping, identifying the vertices of polygons as they are reflected, rotated, and translated);
- and determining horizontal and vertical distances between points on a coordinate grid in the first quadrant.

10. Demonstrates conceptual understanding of spatial reasoning and visualization from two- or three-dimensional representations by building models of :

- rectangular and triangular prisms
- cones
- cylinders
- pyramids

Strand 3 - Functions and Algebra

1. Identifies and extends to specific cases a variety of patterns (linear and nonlinear) represented in models, tables, sequences, or in problem situations; and writes a rule in words or symbols for finding specific cases of a linear relationship.

2. Demonstrates conceptual understanding of linear relationships ($y = kx$) as a constant rate of change by identifying, describing, or comparing situations that represent constant rates of change (e.g., tell a story given a line graph about a trip).

3. Demonstrates conceptual understanding of algebraic expressions by:

- using letters to represent unknown quantities to write linear algebraic expressions involving any two of the four operations;
- evaluating linear algebraic expressions using whole numbers.

4. Demonstrates conceptual understanding of equality by:

- showing equivalence between two expressions using models or different representations of the expressions
- solving one-step linear equations of the form $ax = c$, $x \pm b = c$, or $x/a = c$, where a, b, and c are whole numbers with $a \neq 0$;

- determining which values of a replacement set make the equation (multi-step of the form $ax \pm b = c$ where a, b, and c are whole numbers with $a \neq 0$) a true statement (e.g., $2x + 3 = 11$, $\{x: x = 2, 3, 4, 5\}$).

Strand 4 - Data, Statistics, and Probability

1. Interprets a given representation (tables, bar graphs, circle graphs, or line graphs) to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.

2. Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using measures of central tendency (mean, median, or mode) or range to analyze situations, or to solve problems.

3. Organizes and displays data using tables, bar graphs, or line graphs to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems:

- identifie or describes representations or elements of representations that best display a given set of data or situation, consistent with the given representations required.

4. No standard at this grade.

5. For a probability event in which the sample space may or may not contain equally likely outcomes:

- **predicts** the likelihood of an event as a fraction and tests the prediction through experiments; and determines if a game is fair.
- **determines** the experimental or theoretical probability of an event and expresses the result as a fraction.

6. In response to a teacher or student generated question or hypothesis:

- decides the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical) necessary to answer the question;
- collects, organizes, and appropriately displays the data;
- analyzes the data to draw conclusions about the question or hypothesis being tested,
- makes predictions when appropriate;
- asks new questions
- makes connections to real world situations.

Process Strands

Strand 5: Problem Solving, Reasoning, and Proof

1. Students will use problem-solving strategies to investigate and understand increasingly complex mathematical content and be able to:

- determine the reasonableness of solutions to real-world problems.
- generalize solutions and apply strategies to new problem situations.
- add to the repertoire of problem-solving strategies (e.g., looking for similar problems) and use those strategies in more sophisticated ways.
- solve problems with multiple solutions, recognize when a problem has no solution, and recognize problems where more information is needed.
- translate results of a computation into solutions that fit the real-world problem (e.g., when a computation shows that one needs 3.2 gallons of paint to paint a room, how much paint do you buy?).

2. Students will use mathematical reasoning and proof and be able to:

- draw conclusions and solve problems using elementary deductive reasoning and reasoning by analogy.
- make and defend conjectures and generalizations.
- use models, known facts, properties, and relationships to explain thinking and to justify answers and solution processes.
- recognize the pervasive use and power of reasoning as a part of mathematics.

Strand 6: Communication, Connections, and Representations

1. Students will communicate their understanding of mathematics and be able to:

- discuss mathematical ideas and write convincing arguments.
- understand, explain, analyze, and evaluate mathematical arguments and conclusions presented by others.
- ask clarifying and extending questions related to mathematics they have heard or read about.
- understand and appreciate the economy and power of mathematical symbolism and its role in the development of mathematics.
- demonstrate an understanding of mathematical concepts and relationships through a variety of methods (e.g., writing, graphing, charts, diagrams, number sentences, or symbols).
- use a variety of technologies (e.g., computers, calculators, video, probes) to represent and communicate mathematical ideas.

2. Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:

- use physical models and diagrams to represent important mathematical ideas (e.g., multiplication).
- use appropriate representations to solve problems or to portray, clarify, or extend a mathematical idea.
- recognize equivalent representations of concepts and procedures and translate among them as appropriate (for example, understand how the addition of whole numbers, fractions, and decimals are related).

3. Students will recognize, explore, and develop mathematical connections and be able to:

- see mathematics as an integrated whole.
- recognize relationships among different topics in mathematics.
- recognize and use mathematics in other curriculum areas and in their daily lives.
- link concepts and procedures.
- use mathematical skills, concepts, and applications in other disciplines (e.g., graphs in social studies, patterns in art, or music and geometry in technology education).

Grade 6

Content Strands

Strand 1 - Numbers and Operations

1. Demonstrates conceptual understanding of rational numbers using models, explanations, or other representations:

- with respect to ratios (comparison of two whole numbers by division a/b, a:b, a÷b, where b≠0
- rates (e.g., a out of b, 25%)
- ✔ given a decimal representation in hundredths, write an equivalent fraction
- ✔ read and write rational numbers to millionths

2. Demonstrates understanding of the relative magnitude of numbers by ordering or comparing:

- order or compare numbers with whole number bases and whole number exponents, integers, or rational numbers within and across number formats (fractions, decimals, or whole number percents from 1-100)
- ✔ demonstrates conceptual understanding of integers
- ✔ read and write integers and positive rational numbers
- ✔ use models to represent integers
- ✔ recognize absolute value
- ✔ investigate patterns involving the number line, integers and positive rational numbers

3. Demonstrates conceptual understanding of mathematical operations:

- adding and subtracting positive fractions and integers
- multiplying and dividing fractions and decimals
- describe or illustrate the meaning of a power by representing the relationship between the base (whole number) and the exponent (whole number)
- describe or illustrate the effect on the magnitude of a whole number when multiplying or dividing it by a whole number, decimal, or fraction.
- use the order operations to evaluate expressions (for example: multiply/divide and add/subtract)
- ✔ develop and use algorithms to add, subtract, and multiply decimals
- ✔ introduce the division of two-and three-digit decimals
- ✔ demonstrate knowledge of inverse relationships among the four basic operations as applied to positive rational numbers

4. Accurately solves problems:

- involving single or multiple operations on fractions (proper, improper, and mixed) or decimals
- involving addition or subtraction of integers
- involving percent of a whole
- problems involving greatest common factor or least common multiple

5. No Standard at this grade.

6. Uses a variety of mental computation strategies to solve problems and to determine the reasonableness of answers and mentally calculates:

- determines the part of a whole number using benchmark percents (1%, 10%, 25%, 50%, and 75%)

7. Makes estimates in a given situation by identifying when estimation is appropriate:

- select the appropriate method of estimation
- determine the level of accuracy needed given the situation
- analyze the effect of the estimation method on the accuracy of results
- evaluate the reasonableness of solutions

8. Applies properties of numbers and field properties to solve problems and to simplify computations:

- properties of numbers to be applied are odd, even, positive, negative, remainders, divisibility, and prime factorization.
- field properties to be applied include commutative, associative, identity, multiplicative property of one, distributive and additive inverses.
- ✔ identify and use prime and composite numbers
- ✔ generate properties and relationships related to prime numbers, composite numbers, rational numbers, multiples, and factors.
- ✔ demonstrate the relationship between fractions and decimals and percent

Strand 2 -- Geometry and Measurement

1. Uses properties or attributes of angles or sides to identify, describe, classify, or distinguish among different types of triangles and quadrilaterals and other shapes:

- attributes of angles to consider are acute, right, and obtuse
- attributes of sides to consider are congruent sides, parallelism, or perpendicularity
- types of triangles to identify are right, acute, obtuse, equiangular, scalene, isosceles, or equilateral
- types of quadrilaterals to identify are rectangles, squares, rhombi, trapezoids, parallelograms
- ✓ identify, describe and name properties of other polygons

2. No standard at this grade.

3. Uses properties or attributes to identify, compare, or describe three-dimensional shapes:

- properties to consider are shape of bases, number of lateral faces, number of bases, number of edges, or number of vertices)
- shapes to consider are rectangular prisms, triangular prisms, cylinders, spheres, pyramids, or cones.

4. Demonstrates conceptual understanding of congruency:

- predict and describe the transformational steps (reflections, translations, and rotations) needed to show congruence (including the degree of rotation)
- use technology, concrete materials, an/or coordinate geometry to explain properties of transformations
- predict and describe the result of composing and decomposing two and three dimensional objects using models or explanations
- use line and rotational symmetry to demonstrate congruent parts within a shape.

5. Demonstrates conceptual understanding of similarity:

- describe the proportional effect on the linear dimensions of polygons and circles when scaling up or down while preserving the angles of polygons
- solve related problems including applying scales on maps
- describes effects using models or explanations.
- compare the relationship among similar figures and their areas

6. Demonstrates conceptual understanding of perimeter of polygons, area and volume:

- perimeter of polygons
- area of quadrilaterals or triangles
- volume of rectangular prisms by using models, formulas, or by solving problems.
- demonstrates understanding of the relationships of circle measures (radius to diameter and diameter to circumference) by solving related problems.
- expresses all measures using appropriate units.
- ✓ solve problems involving two- and three-dimensional figures

- ✓ given a standard unit, estimate the area of any region
- ✓ find and/or estimate the perimeter and area of a given quadrilateral or triangle
- ✓ approximate areas of irregular figures drawn on a grid
- ✓ determine how a change in length or width affects perimeter and area

7. Measures and uses units of measures appropriately and consistently, and makes conversions within systems when solving problems across the content strands.

Measurement Benchmarks

Measures:	Grade 6
Length	**Units (accuracy):** Inch (to 1/8 inch); Foot; Centimeter (to 0.5 centimeter); Meter (to 0.5 centimeter); Yard; Mile (use in scale questions); Kilometer (use in scale questions) **Equivalencies:** 12 inches in 1 foot; 100 centimeters in 1 meter; 3 feet in 1 yard; 36 inches in 1 yard; 10 millimeters in 1 centimeter
Time	**Unit (accuracy):** Hour (to 1 minute); Day; Year **Equivalencies:** 24 hours in 1 day; 7 days in 1 week; 365 days in 1 year; 60 seconds in 1 minute; 60 minutes in 1 hour
Temperature	**Unit (accuracy):** Cº and Fº (to 1 degree)
Capacity	**Unit (accuracy):** Quart (to 1 ounce); Gallon; Pint **Equivalencies:** 32 ounces in 1 quart; 4 quarts in 1 gallon; 2 pints in 1 quart
Mass	**Unit (accuracy):** Kilogram; Gram (to whole gram)
Weight	**Unit (accuracy):** Pound (to 1 ounce) **Equivalencies:** 16 ounces in 1 pound
Angles and Rotation	**Unit (accuracy):** Degree (to 2 degrees) **Equivalencies:** 360^0 in 1 circle; 90^0 in 1 right angle

8. No standard at this grade.

9. No standard at this grade.

10. No standard at this grade.

Strand 3 -- Functions and Algebra

1. Identifies and extends to specific cases a variety of patterns, linear and nonlinear:

- identify patterns represented in models, tables, sequences, graphs, or in problem situations.
- write a rule in words or symbols for finding specific cases of a linear relationship
- write a rule in words or symbols for finding specific cases of a nonlinear relationship

- write an expression or equation using words or symbols to express the generalization of a linear relationship (e.g. twice the term number plus 1 or $2n + 1$)
- ✓ solve simple linear equations using concrete, informal methods

2. Demonstrates conceptual understanding of linear relationships (y = kx; y = mx + b) as a constant rate of change:

- construct and interpret graphs of real occurrences
- describe the slope of linear relationships (faster, slower, greater, or smaller) in a variety of problems
- describe how change in the value of one variable relates to change in the value of a second variable in problem situations with constant rates of change.

3. Demonstrates conceptual understanding of algebraic expressions:

- use letters to represent unknown quantities to write linear algebraic expressions involving any of the four operations and consistent with order of operations expected at this grade level.
- evaluate linear algebraic expressions
- evaluate an expression within an equation (e.g. determine the value of y when $x = 4$ given $y = 3x - 2$)
- ✓ understand meaning of variable, expression and equation

4 Demonstrates conceptual understanding of equality:

- show equivalence between two expressions using models or different representations of the expressions.

Strand 4 -- Data, Statistics and Probability

1. Interprets a given representation:

- interpret circle graphs, line graphs, or stem-and-leaf plots to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.

2. Analyzes patterns, trends or distributions in data in a variety of contexts:

- determine and use measure of central tendency (mean, median, or mode) or dispersion (range) to analyze situations, or to solve problems.

3. Organizes and displays data:

- use tables, line graphs, or stem and leaf plots to answer questions related to the data.
- analyze data to formulae or justify conclusions, to make predictions, or to solve problems.

4. Uses counting techniques to solve problems:

- solve problems involving combinations or simple permutations using a variety of strategies (e.g. organized lists, tables, tree diagrams, models, *Fundamental Counting Principle*, or others).
  - ✓ uses sampling techniques to make predictions.

5. For a probability event in which the sample space may or may not contain equally likely outcomes:

- predicts the theoretical probability of an event and tests the prediction through experiments and simulations; and designs fair games.
- determines the experimental or theoretical probability of an event in a problem solving situation.

6. In response to a teacher or student generated question or hypothesis decides the most effective method to collect data necessary to answer the questions:

- collects, organizes, and appropriately displays the data
- analyzes the data to draw conclusions about the question or hypothesis being tested
- makes appropriate predictions and asks new questions
- makes connections to real world situations.

Process Strands

Strand 5: Problem Solving, Reasoning, and Proof

1. Students will use mathematical reasoning and proof:

- draw logical conclusions and make generalizations using deductive and inductive reasoning
- formulate, test, and justify mathematical conjectures and arguments
- construct and determine the validity of a mathematical argument or a solution
- apply mathematical reasoning skills in other disciplines
  - ✓ solve problems using a variety of strategies
  - ✓ formulate and solve real-world problems
  - ✓ solve multi-step problems and problems with multiple solutions or no solution, and recognize problems where more information is needed.
  - ✓ demonstrate that a problem may be solved in more than one way
  - ✓ generalize solutions and apply strategies to new problem situations
  - ✓ develop confidence, perseverance, and persistence in problem solving both independently and in groups
  - ✓ use technology wherever appropriate to solve real-world problems

Appendix B: Classification of Word Problems[1]

Classification of word problems: Researchers have identified four basic categories of addition and subtraction problems: problems with joining actions, separating actions, part-part whole relationships, and comparison situations. Table 1 contains

some examples to illustrate the distinctions in the categories identified by the researchers, but there may be additional ways to express the same actions, relationships, or situations. For example, the following are additional ways0 to ask questions related to the "Join – Start Unknown" category.

Example 1: "I am thinking of a number that when you add 5 to it, the answer is 13. What number am I thinking of?"
Example 2: John puts a five-pound toy inside an empty wooden box. The box and toy together weigh 7 pounds. How much does the empty box weigh?

In summary, students should have experiences with problem solving in addition and subtraction across a variety of problem types. It is important to remember that any problem situation that fits the equations given and the category can be asked, not just problems as stated in Table 1.

 "Teaching students to add and subtract involves providing students with an opportunity to explore a rich set of problems with different semantic structures and to develop a variety of strategies to quantify, represent, calculate, express, and justify results." (Gutstein, E., Romberg, 1995)

Appendix A, NH State Frameworks, Table 1

Join	(Result Unknown) Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have altogether? $$5 + 8 = \square$$	(Change unknown) Connie has 5 marbles. How many more marbles does she need to have 13 marbles altogether? $$5 + \square = 13$$	(Start Unknown) Connie has some marbles. Juan gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with? $$\square + 5 = 13$$
Separate	(Result Unknown) Connie had 13 marbles. She gave 5 marbles to Juan. How many marbles does she have left? $$13 - 5 = \square$$	(Change unknown) Connie has 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give Juan? $$13 - \square = 5$$	(Start Unknown) Connie has some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to start with? $$\square - 5 = 8$$

Part-Part-Whole	(Whole Unknown) Connie has 5 red marbles and 8 blue marbles. How many marbles does Connie have? $5 + 8 = \square$	(Part Unknown) Connie has 13 marbles. All the marbles are either blue or red. Connie has 5 red marbles. How many blue marbles does Connie have? $13 - 5 = \square$	
Compare	(Difference Unknown) Connie has 13 marbles. Juan has 5 marbles. How many more marbles does Connie have than Juan? $13 - 5 = \square$	(Compare Quantity Unknown) Juan has 5 marbles. Connie has 8 more marbles than Juan. How many marbles does Connie have? $5 + 8 = \square$	(Referent Unknown) Connie has 13 marbles. She has 5 more marbles than Juan. How many marbles does Juan have? $13 - 5 = \square$

[1] Carpenter, Fennema, Peterson, Chiang, and Loef (1989) cited in Gutstein, E., Romberg, T., *Teaching Children to Add and Subtract*, Journal of Mathematical Behavior, 14, 283-324 (1995).

Strand 6 - Communication, Connections, and Representations

1. Students will communicate their understanding of mathematics and be able to:

- articulate ideas clearly and logically in both written and oral form
- present, share, explain, and justify thinking with others to solve problems
- use mathematical symbols and notation
- formulate questions, conjectures, definitions and generalizations about data, information, and problem situations

2. Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:

- use models and technology to develop equivalent representations of the same mathematical concept.
- use and create representations to solve problems and organize their thoughts and ideas.
- convert between representations (e.g. a table of values, an equation, and a graph may all be representations of the same function.)
- ✓ use a variety of technologies to represent and communicate mathematical ideas
- ✓ understand the mathematical processes and procedures of addition, subtraction, multiplication, and division and relate them to one another

3. **Students will recognize, explore, and develop mathematical connections and be able to:**

- connect new mathematical ideas to those already studied and build upon them
- understand that many real-world applications require an understanding of mathematical concepts (e.g. personal finance, running a business, building a house, following a recipe, or sending a rocket to the moon)
- explain in oral and written form the relationships between a real-world problem and an appropriate mathematical model.
- explain in oral and written form the relationships among various mathematical concepts (e.g. the relationship between exponentiation and multiplication)
- ✓ apply math skills in other disciplines

Grade 7

Content Strands

Strand 1 -- Numbers and Operations

1. **Demonstrates conceptual understanding of rational numbers:**
   - square roots of perfect squares
   - rates
   - proportional reasoning

2. **Demonstrates conceptual understanding of rational numbers with respect to the following using models, explanations, or other representations:**

   - ✓ percents as a means of comparing the same or different parts of the whole when the wholes
   - ✓ vary in magnitude (e.g. 8 girls in a classroom of 16 students compared to 8 girls in a classroom of 20 students 20% of 400 compared to 50% or 100
   - ✓ percents as a way of expressing multiples of a number (e.g.., 200% of 50)

3. Demonstrates understanding of the relative magnitude of numbers by ordering, comparing, or identifying equivalent rational numbers across number formats numbers with whole number bases and whole number exponents (e.g., 3^3, 4^3) integers:

- ✓ absolute values
- ✓ numbers represented in scientific notation using number lines or equality and inequality symbols.

4. Demonstrates conceptual understand of operations with the following using models, diagrams, or explanations

- integers and whole number exponents (where the base is a whole number)

5. Accurately solves problems involving:

- ✓ proportional reasoning
- ✓ percents involving discounts, tax or tips
- ✓ rates

(IMPORTANT: Applies the conventions of order of operations including parentheses, brackets or exponents.)

6. No standard at this grade.

7. Uses a variety of mental computation strategies to solve problems:

- (e.g., using compatible numbers, applying properties of operations, using mental imagery, using patterns) and to determine the reasonableness of answers; and mentally calculates
- <u>benchmark perfect squares and related square roots</u> (e.g., 1^2, 2^2, …, 12^2, 15^2, 20^2, 25^2, 100^2, 1000^2)
- determines the <u>part of a number</u> using benchmark percents and <u>related fractions</u> (1%, 10%, 25%, $33\frac{1}{3}$%, 50%, $66\frac{2}{3}$% , 75%, and 100%) (e.g. 25% of 16; $33\frac{1}{3}$% of 330).

  (IMPORTANT: (The intent of this proficiency is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.)

8. Makes estimates in a given situation:

- (<u>including tips, discounts, and tax</u>) by identifying when estimation is appropriate
- selecting the appropriate method of estimation
- determining the level of accuracy needed given the situation
- analyzing the effect of the estimation method on the accuracy of results
- evaluating the reasonableness of solutions appropriate to grade level proficiencies across content strands.

(IMPORTANT: (The intent of this proficiency is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.)

9. Applies properties of numbers (odd, even, positive, negative, remainders, divisibility, and prime factorization) and field properties (commutative, associative, identity, distributive, inverses) to solve problems and to simplify computations, and demonstrate conceptual understanding of field properties:

- as they apply to subsets of the real numbers (e.g., the set of whole numbers does not have additive inverses, the set of integers does not have multiplicative inverses).

Strand 2 - -Geometry and Measurement

1. Uses properties of angle relationships

- ✓ resulting from two or three intersecting lines (adjacent angles, vertical angles, straight angles,
- ✓ or angle relationships formed by two non-parallel lines cut by a transversal) formed by two non-parallel lines cut by a transversal) or two parallel lines cut by a transversal to solve problems

2. Applies theorems or relationships

- ✓ (triangle inequality or sum of the measures of interior angles of regular polygons) to solve problems

3. No standard at this grade.

4. Applies the concepts of congruency:

- ✓ by solving problems on a coordinate plane involving reflections, translations, or rotations

5. Applies concepts of similarity by:

- ✓ solving problems involving scaling up or down and their impact on angle measures
- ✓ linear dimensions and areas of polygons, and circles when the linear dimensions are multiplied by a constant factor
- ✓ Describes effects using models or explanations

6. Demonstrates conceptual understanding using models, formulas, or by solving related problems for:

- ✓ area of circles or the area or perimeter of composite figures (quadrilaterals, triangles, or parts of circles),
- ✓ surface area of rectangular prisms,
- ✓ volume of rectangular prisms, triangular prisms, or cylinders
- ✓ Expresses all measures using appropriate units

Measurement Benchmarks

Measures:	Grade 7
Length	**Units (accuracy):** Inch (to 1/8 inch); Foot; Centimeter (to 0.5 centimeter); Meter (to 0.5 centimeter); Yard; Mile (use in scale questions); Kilometer (use in scale questions) **Equivalencies:** 12 inches in 1 foot; 100 centimeters in 1 meter; 3 feet in 1 yard; 36 inches in 1 yard; 10 millimeters in 1 centimeter
Time	**Unit (accuracy):** Hour (to 1 minute); Day; Year **Equivalencies:** 24 hours in 1 day; 7 days in 1 week; 365 days in 1 year; 60 seconds in 1 minute; 60 minutes in 1 hour
Temperature	**Unit (accuracy):** Cº and Fº (to 1 degree)
Capacity	**Unit (accuracy):** Quart (to 1 ounce); Gallon; Pint **Equivalencies:** 32 ounces in 1 quart; 4 quarts in 1 gallon; 2 pints in 1 quart
Mass	**Unit (accuracy):** Kilogram; Gram (to whole gram)
Weight	**Unit (accuracy):** Pound (to 1 ounce) **Equivalencies:** 16 ounces in 1 pound
Angles and Rotation	**Unit (accuracy):** Degree (to 2 degrees) **Equivalencies:** 360^0 in 1 circle; 90^0 in 1 right angle

7. No standard at this grade.

8. No standard at this grade.

9. No standard at this grade.

10. Demonstrates conceptual understanding of spatial reasoning and visualization by:

- sketching three-dimensional solids
- draws nets of rectangular and triangular prisms, cylinders, and pyramids
- uses the nets as a technique for finding surface area

Strand 3 –Functions and Algebra

1. Identifies and extends to specific cases a variety of linear and non-linear patterns:

- ✓ represented in models, tables, sequences, graphs, or in problem situations

- ✓ generalizes a linear relationship using words and symbols;
- ✓ generalizes a linear relationship to find a specific case
- ✓ writes an expression or equation using words or symbols to express the generalization of a nonlinear relationship

2. Demonstrates conceptual understanding of linear relationships ($y = kx$; $y = mx + b$) as a constant rate of change by:

- ✓ solving problems involving the relationship between slope and rate of change
- ✓ describing the meaning of slope in concrete situations
- ✓ informally determining the slope of a line from a table or graph

and:
- ✓ distinguishes between constant and varying rates of change in concrete situations represented in tables or graphs
- ✓ describes how change in the value of one variable relates to change in the value of a second variable in problem situations with constant rates of change.

3. Demonstrates conceptual understanding of algebraic expressions:

- ✓ by using letters to represent unknown quantities to write algebraic expressions (including those with whole number exponents or more than one variable);
- ✓ evaluating algebraic expressions (including those with whole number exponents or more than one variable);
- ✓ evaluating an expression within an equation (e.g., determine the value of y when x = 4 given $y = 5x3 – 2$).

4. Demonstrates conceptual understanding of equality:

- ✓ by showing equivalence between two expressions (expressions consistent with the parameters of the left- and right-hand sides of the equations being solved at this grade level) using models or different representations of the expressions
- ✓ solving multi-step linear equations of the form $ax \pm b = c$ with $a \neq 0$, $ax \pm b = cx \pm d$ with a, c $\neq$ 0, and $(x/a) \pm b = c$ with $a \neq 0$, where a, b, c and d are whole numbers
- ✓ translating a problem-solving situation into an equation consistent with the parameters of the type of equations being solved for this grade level.

Strand 4 --Data, Statistics and Probability

1. Interprets a given representation:

- ✓ (circle graphs, scatter plots that represent discrete linear relationships, or histograms) to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.

  (IMPORTANT: Analyzes data consistent with concepts and skills in Data, Statistics and Probability # 2)

2. Analyzes patterns, trends, or distributions in data in a variety of contexts by solving problems:

✓ using measures of central tendency (mean, median, or mode), dispersion (range or variation), or outliers to analyze situations to determine their effect on mean, median, or mode; and evaluates the sample from which the statistics were developed (bias).

3. Organizes and displays data:

- using tables, line graphs, scatter plots, and circle graphs to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.

7. Identifies or describes representations or elements of representations that best display a given set of data or situation:

✓ consistent with the representations required in Data, Statistics and Probability # 1.

(IMPORTANT: Analyzes data consistent with concepts and skills in Data, Statistics and Probability # 2)

5. Uses counting techniques to solve problems:

- in context involving combinations or permutations (e.g., How many different ways can eight students place first, second, and third in a race?) using a variety of strategies (e.g., organized lists, tables, tree diagrams, models, Fundamental Counting Principle, or[sc] others).

6. For a probability event in which the sample space may or may not contain equally likely outcomes, predicts:

- the theoretical probability of an event and tests the prediction through experiments and simulations; and compares and contrasts theoretical and experimental probabilities.

7. For a probability event in which the sample space may or may not contain equally   likely outcomes, determines:

✓ the experimental or theoretical probability of an event in a problem-solving situation.

8. **In response to a teacher or student generated question or hypothesis:**

- decides the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical)
- necessary to answer the question;
- collects,
- organizes
- appropriately displays the data;
- analyzes the data to draw conclusions about the question or hypothesis being tested while considering the limitations that could affect interpretations;
- when appropriate makes predictions; and asks new questions and makes connections to real world situations.

(IMPORTANT: Analyzes data consistent with concepts and skills in Data, Statistics and Probability # 2)

Process Strands

Strand 5 -- Problem Solving, Reasoning, and Proof

1. **Students will use problem-solving strategies to investigate and understand increasingly complex mathematical content and be able to:**

- use problem-solving strategies appropriately and effectively for a given situation.

- determine, collect and organize the relevant information needed to solve real-world problems.
- apply integrated problem-solving strategies to solve problems in the physical, natural, and social sciences and in pure mathematics.
- use technology when appropriate to solve problems.
- reflect on solutions and the problem-solving process for a given situation and refine strategies as needed.

2. Students will use mathematical reasoning and proof and be able to:

- draw logical conclusions and make generalizations using deductive and inductive reasoning.
- formulate, test, and justify mathematical conjectures and arguments.
- construct and determine the validity of a mathematical argument or a solution.
- apply mathematical reasoning skills in other disciplines.

Appendix B: Classification of Word Problems[1]

Classification of word problems: Researchers have identified four basic categories of addition and subtraction problems: problems with joining actions, separating actions, part-part whole relationships, and comparison situations. Table 1 contains some examples to illustrate the distinctions in the categories identified by the researchers, but there may be additional ways to express the same actions, relationships, or situations. For example, the following are additional ways0 to ask questions related to the "Join – Start Unknown" category.

Example 1: "I am thinking of a number that when you add 5 to it, the answer is 13. What number am I thinking of?"
Example 2: John puts a five-pound toy inside an empty wooden box. The box and toy together weigh 7 pounds. How much does the empty box weigh?

In summary, students should have experiences with problem solving in addition and subtraction across a variety of problem types. It is important to remember that any problem situation that fits the equations given and the category can be asked, not just problems as stated in Table 1.

 "Teaching students to add and subtract involves providing students with an opportunity to explore a rich set of problems with different semantic structures and to develop a variety of strategies to quantify, represent, calculate, express, and justify results." (Gutstein, E., Romberg, 1995)

Appendix A, NH State Frameworks, Table 1

Join	(Result Unknown) Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have altogether? $5 + 8 = \square$	(Change unknown) Connie has 5 marbles. How many more marbles does she need to have 13 marbles altogether? $5 + \square = 13$	(Start Unknown) Connie has some marbles. Juan gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with? $\square + 5 = 13$
Separate	(Result Unknown) Connie had 13 marbles. She gave 5 marbles to Juan. How many marbles does she have left? $13 - 5 = \square$	(Change unknown) Connie has 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give Juan? $13 - \square = 5$	(Start Unknown) Connie has some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to start with? $\square - 5 = 8$
Part-Part-Whole	(Whole Unknown) Connie has 5 red marbles and 8 blue marbles. How many marbles does Connie have? $5 + 8 = \square$	(Part Unknown) Connie has 13 marbles. All the marbles are either blue or red. Connie has 5 red marbles. How many blue marbles does Connie have? $13 - 5 = \square$	
Compare	(Difference Unknown) Connie has 13 marbles. Juan has 5 marbles. How many more marbles does Connie have than Juan? $13 - 5 = \square$	(Compare Quantity Unknown) Juan has 5 marbles. Connie has 8 more marbles than Juan. How many marbles does Connie have? $5 + 8 = \square$	(Referent Unknown) Connie has 13 marbles. She has 5 more marbles than Juan. How many marbles does Juan have? $13 - 5 = \square$

[1] Carpenter, Fennema, Peterson, Chiang, and Loef (1989) cited in Gutstein, E., Romberg, T., *Teaching Children to Add and Subtract*, Journal of Mathematical Behavior, 14, 283-324 (1995).

Strand 6 -- Communication, Connections, and Representations

1. **Students will communicate their understanding of mathematics and be able to:**

- articulate ideas clearly and logically in both written and oral form.
- present, share, explain, and justify thinking with others and build upon the ideas of others to solve problems.
- use mathematical symbols and notation.
- formulate questions, conjectures, definitions, and generalizations about data, information, and problem situations.

2. Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:

- use models and technology to develop equivalent representations of the same mathematical concept.
- use and create representations to solve problems and organize their thoughts and ideas.
- convert between representations (e.g., a table of values, an equation, and a graph may all be representations of the same function).

3. Students will recognize, explore, and develop mathematical connections and be able to:

- connect new mathematical ideas to those already studied and build upon them.
- understand that many real-world applications require an understanding of mathematical concepts (e.g., personal finance, running a business, building a house, following a recipe, or sending a rocket to the moon).
- explain in oral and written form the relationships between a real-world problem and an appropriate mathematical model.
- explain in oral and written form the relationships among various mathematical concepts (e.g., the relationship between exponentiation and multiplication).

Grade 8

Content Strands

Standard 1 - Number and Operations

1. **Demonstrates conceptual understanding of rational numbers with respect to the following using explanations, models, or other representations:**

   - absolute values
   - perfect square and cube roots
   - percents as a way of describing change (percent increase and decrease).

2. **Demonstrates understanding of the relative magnitude of numbers by:**

   - ordering or comparing rational numbers
   - common irrational numbers (e.g., $\sqrt{2}$, π)
   - numbers with whole number or fractional bases and whole number exponents
   - square roots
   - absolute values
   - integers
   - numbers represented in scientific notation using number lines or equality and inequality

3. **No standard at this grade.**

4. **Accurately solves problems involving:**

- proportional reasoning (percent increase or decrease, interest rates, markups, or rates)
- multiplication or division of integers
- squares
- cubes
- taking square or cube roots

5. No standard at this grade.

Strand 2 -- Geometry and Measurement

1. No standard at this grade.

2. Applies the Pythagorean Theorem:

- to find a missing side of a right triangle
- in problem solving situations

3. No standard at this grade.

4. No standard at this grade

5. Applies concepts of similarity:

- to determine the impact of scaling on the volume or surface area of three-dimensional figures when linear dimensions are multiplied by a constant factor
- to determine the length of sides of similar triangles
- to solve problems involving growth and rate

6. Demonstrates conceptual understanding of surface area or volume by solving problems involving surface area and volume of:

- rectangular prisms
- triangular prisms
- cylinders
- pyramids
- cones

7. No standard at this grade

Measurement Benchmarks

Measures	Grade 8

Length	**Units (accuracy):** Inch (to 1/16 inch); Foot; Centimeter (to 1/10 centimeter); Meter (to 1/100 meter); Yard; Mile (use in scale and rate questions); Kilometer (use in scale and rate questions) **Equivalencies:** 12 inches in 1 foot; 100 centimeters in 1 meter; 3 feet in 1 yard; 36 inches in 1 yard; 10 millimeters in 1 centimeter; 1000 millimeters in 1 meter
Time	**Unit (accuracy):** Hour (to 1 minute); Day; Year **Equivalencies:** 24 hours in 1 day; 7 days in 1 week; 365 days in 1 year; 60 seconds in 1 minute; 60 minutes in 1 hour
Temperature	**Unit (accuracy):** Cº and Fº (to 1 degree)
Capacity	**Unit (accuracy):** Quarts (to 1 ounce); Gallon; Pint; Liter **Equivalencies:** 32 ounces in 1 quart; 4 quarts in 1 gallon; 2 pints in 1 quart; 1000 milliliters in 1 liter
Mass	**Unit (accuracy):** Kilogram; Gram (to 1/10 gram)
Weight	**Unit (accuracy):** Pound (to 1 ounce) **Equivalencies:** 16 ounces in 1 pound
Angles and Rotation	**Unit (accuracy):** Degree (to 2 degrees) **Equivalencies:** 360º in 1 circle; 90º in 1 right angle

8. No standard at this grade.

9. No standard at this grade.

10. No standard at this grade.

Strand 3 -- Functions and Algebra

1. **Identifies and extends to specific cases a variety of patterns (linear and nonlinear):**

   - represented in models, tables, sequences, graphs, or in problem situations
   - generalizes a linear relationship (non-recursive explicit equation)
   - generalizes a linear relationship to find a specific case
   - generalizes a nonlinear relationship using words or symbols
   - generalizes a common nonlinear relationship to find a specific case

2. Demonstrates conceptual understanding of linear relationships ($y = kx$; $y = mx + b$) as a constant rate of change:

   - by solving problems involving the relationship between slope and rate of change
   - informally and formally determining slopes and intercepts represented in graphs, tables, or problem situations
   - by describing the meaning of slope and intercept in context

- by distinguishing between linear relationships (constant rates of change) and nonlinear relationships (varying rates of change) represented in tables graphs, equations, or problem situations
- by describing how change in the value of one variable relates to change in the value of a second variable in problem situations with constant and varying rates of change.

3. Demonstrates conceptual understanding of algebraic expressions:

- by evaluating and simplifying algebraic expressions (including those with square roots, whole number exponents, or rational numbers)
- by evaluating an expression within an equation (e.g., determine the value of y when $x = 4$ given $y = 7\sqrt{x} + 2x$).

4. Demonstrates conceptual understanding of equality:

- by showing equivalence between two expressions (expressions consistent with the parameters of the left- and right-hand sides of the equations being solved at this grade level) using models or different representations of the expressions,
- by solving formulas for a variable requiring one transformation (e.g., $d = rt$; $d/r = t$)
- by solving multi-step linear equations with integer coefficients
- by showing that two expressions are or are not equivalent by applying commutative, associative, or distributive properties, order of operations, or substitution by informally solving problems involving systems of linear equations in a contex

Strand 4 -- Data, Statistics, and Probability

1. Interprets a given representation (line graphs, scatter plots, histograms, or box-and-whisker plots):
- to analyze the data to formulate or justify conclusions
- to make predictions
- to solve problems

2. Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using:

- measures of central tendency (mean, median, or mode)
- dispersion (range or variation)
- outliers
- quartile values
- estimated line of best fit to analyze situations, or to solve problems; and evaluates the sample from which the statistics were developed (bias, random, or non-random)

3. **Organizes and displays data:**

- using scatter plots to answer questions related to the data

- to analyze the data to formulate or justify conclusions
- to make predictions
- to solve problems; or identifies representations or elements of representations that best display a given set of data or situation

4. **Uses counting techniques to solve problems in context involving combinations or permutations using:**

- organized lists
- tables
- tree diagrams
- models
- Fundamental Counting Principle

5. For a probability event in which the sample space may or may not contain equally likely outcomes:

- determines the experimental or theoretical probability of an event in a problem-solving situation
- predicts the theoretical probability of an event and tests the prediction through experiments and simulations
- compares and contrasts theoretical and experimental probabilities.

6. **In response to a teacher or student generated question or hypothesis:**

- decides the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical) necessary to answer the question
- collects, organizes, and appropriately displays the data
- analyzes the data to draw conclusions about the question or hypothesis being tested while considering the limitations that could affect interpretations
- makes predictions when appropriate
- asks new questions and makes connections to real world situations

Process Strands

Strand 5 -- Problem Solving, Reasoning, and Proof

1. Students will use problem-solving strategies to investigate and understand increasingly complex mathematical content and be able to:

- **Use problem-solving strategies appropriately and effectively for a given situation.**
- Determine, collect and organize the relevant information needed to solve real-world problems.
- Apply integrated problem-solving strategies to solve problems in the physical, natural, and social sciences and in pure mathematics.
- Use technology when appropriate to solve problems.
- Reflect on solutions and the problem-solving process for a given situation and refine strategies as needed.

2. Students will use mathematical reasoning and proof and be able to:

- Draw logical conclusions and make generalizations using deductive and inductive reasoning.
- Formulate, test, and justify mathematical conjectures and arguments.
- Construct and determine the validity of a mathematical argument or a solution.
- Apply mathematical reasoning skills in other disciplines.

Appendix B: Classification of Word Problems[1]

Classification of word problems: Researchers have identified four basic categories of addition and subtraction problems: problems with joining actions, separating actions, part-part whole relationships, and comparison situations. Table 1 contains some examples to illustrate the distinctions in the categories identified by the researchers, but there may be additional ways to express the same actions, relationships, or situations. For example, the following are additional ways0 to ask questions related to the "Join – Start Unknown" category.

Example 1: "I am thinking of a number that when you add 5 to it, the answer is 13. What number am I thinking of?"
Example 2: John puts a five-pound toy inside an empty wooden box. The box and toy together weigh 7 pounds. How much does the empty box weigh?
In summary, students should have experiences with problem solving in addition and subtraction across a variety of problem types. It is important to remember that

any problem situation that fits the equations given and the category can be asked, not just problems as stated in Table 1.

"Teaching students to add and subtract involves providing students with an opportunity to explore a rich set of problems with different semantic structures and to develop a variety of strategies to quantify, represent, calculate, express, and justify results." (Gutstein, E., Romberg, 1995)

Appendix A, NH State Frameworks, Table 1

Join	(Result Unknown) Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have altogether? $5 + 8 = \square$	(Change unknown) Connie has 5 marbles. How many more marbles does she need to have 13 marbles altogether? $5 + \square = 13$	(Start Unknown) Connie has some marbles. Juan gave her 5 more marbles. Now she has 13 marbles. How many marbles did Connie have to start with? $\square + 5 = 13$
Separate	(Result Unknown) Connie had 13 marbles. She gave 5 marbles to Juan. How many marbles does she have left? $13 - 5 = \square$	(Change unknown) Connie has 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give Juan? $13 - \square = 5$	(Start Unknown) Connie has some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to start with? $\square - 5 = 8$
Part-Part-Whole	(Whole Unknown) Connie has 5 red marbles and 8 blue marbles. How many marbles does Connie have? $5 + 8 = \square$	(Part Unknown) Connie has 13 marbles. All the marbles are either blue or red. Connie has 5 red marbles. How many blue marbles does Connie have? $13 - 5 = \square$	
Compare	(Difference Unknown) Connie has 13 marbles. Juan has 5 marbles. How many more marbles does Connie have than Juan? $13 - 5 = \square$	(Compare Quantity Unknown) Juan has 5 marbles. Connie has 8 more marbles than Juan. How many marbles does Connie have? $5 + 8 = \square$	(Referent Unknown) Connie has 13 marbles. She has 5 more marbles than Juan. How many marbles does Juan have? $13 - 5 = \square$

[1] Carpenter, Fennema, Peterson, Chiang, and Loef (1989) cited in Gutstein, E., Romberg, T., *Teaching Children to Add and Subtract*, Journal of Mathematical Behavior, 14, 283-324 (1995).

Strand 6 --Communication, Connections, and Representations

1. Students will communicate their understanding of mathematics and be able to:

- Articulate ideas clearly and logically in both written and oral form.
- Present, share, explain, and justify thinking with others and build upon the ideas of others to solve problems.
- Use mathematical symbols and notation.
- Formulate questions, conjectures, definitions, and generalizations about data, information, and problem situations.

2. Students will create and use representations to communicate mathematical ideas and to solve problems and be able to:

- Use models and technology to develop equivalent representations of the same mathematical concept.
- Use and create representations to solve problems and organize their thoughts and ideas.
- Convert between representations (e.g., a table of values, an equation, and a graph may all be representations of the same function).

3. Students will recognize, explore, and develop mathematical connections and be able to:

- Connect new mathematical ideas to those already studied and build upon them.
- Understand that many real-world applications require an understanding of mathematical concepts (e.g., personal finance, running a business, building a house, following a recipe, or sending a rocket to the moon).
- Explain in oral and written form the relationships between a real-world problem and an appropriate mathematical model.
- Explain in oral and written form the relationships among various mathematical concepts (e.g., the relationship between exponentiation and multiplication).

Standards and Proficiencies by Strand

For brevity, *Standards and Proficiencies by Strand* are not included in this working draft. They can be accessed in the shared folder at each school and will be included here in the final document.