

## PRECALCULUS

This course is a systematic development of some of the advanced topics in mathematics for the student with a strong mathematics background. This course is designed to serve as a foundation for the student who expects to enroll in calculus or other college mathematics courses.

### **Content Strand:**

Number and Operations

Geometry and Measurement

Functions and Algebra

Data, Statistics, and Probability

## COURSE COMPETENCIES

This course is an advanced course that extends beyond the States grade span expectations. Those that it does cover are in the advanced category.

1. Solves problems involving complex numbers.
2. Manipulates complex numbers using rectangular and polar coordinates.
3. Finds the  $n$ th roots of complex numbers using De Moivre's Theorem.
4. Derives and uses formulas for length of arcs and area of sectors and area of segments of circles.
5. Uses radian measures appropriately and converts between radian measure and degree measure; and understands why radian measure is useful.
6. Solves problems using analytic geometry and circular trigonometry.
7. Explores and interprets the characteristics of conic sections graphically and algebraically.
8. Identifies, extends, and generalizes a variety of patterns.
9. Computes partial sums of infinite arithmetic sequences, determines when an infinite geometric series converges, and finds its sum. Works between recursive and explicit representations.
10. Identifies arithmetic and geometric sequences and finds the  $n$ th term; then Uses the generalization to find a specific term.
11. Demonstrates conceptual understanding of linear and nonlinear functions and relations
12. Solves equations and verifies/proves identities.
13. Solves equations involving exponential and logarithmic expressions, graphs And interprets the solutions.
14. Uses matrices and determinants to solve systems of equations.
15. Analyzes and interprets measures of dispersion and central tendency.
16. Solves probability problems by applying concepts of counting, random variables, independence/dependence of events, and conditional probabilities.
17. Uses counting techniques to solve problems involving combination or

permutations using a variety of strategies.	
<b>Unit 1:Combinatorics</b>	
<b>Topics</b>	multiplication and additions rules, union and intersection, permutations and combinations
<b>Competencies</b>	17. Uses counting techniques to solve problems involving combination or permutations using a variety of strategies.
<b>Knowledge/Skills</b>	<ul style="list-style-type: none"> <li>• Use Venn diagrams to illustrate intersection and union of sets</li> <li>• Solve counting problems involving intersection and union</li> <li>• Use the multiplication, addition and complement principle to solve counting problems</li> <li>• Solve problems involving combinations and permutations</li> <li>• Solve problems involving repetitive permutations and circular permutations</li> <li>• Use the binomial theorem and Pascal's triangle to solve problems</li> </ul>
<b>Unit 2:Probability</b>	
<b>Topics</b>	dependent and independent events, complements, Binomial Probability Theorem
<b>Competencies</b>	16. Solves probability problems by applying concepts of counting, random variables, independence/dependence of events, and conditional probabilities.
<b>Knowledge/Skills</b>	<ul style="list-style-type: none"> <li>• Find the sample space of an experiment</li> <li>• Find the probability of both dependent and independent events</li> <li>• Use the binomial probability theorem to find the probability of independent repeated trials of a binomial experiment</li> <li>• Solve problems involving conditional probability</li> <li>• Find expected value involving gains and losses</li> </ul>
<b>Unit 3: Statistics</b>	
<b>Topics</b>	Measures of central tendency, measures of dispersion, Normal distributions

<b>Competencies</b>	15. Analyzes and interprets measures of dispersion and central tendency
<b>Knowledge/Skills</b>	<ul style="list-style-type: none"> <li>• Display a set of data using various statistical graphs</li> <li>• Find the mean, median and mode of a set of data</li> <li>• Draw and use box-and-whisker and stem-and-leaf plots</li> <li>• Find the variance and standard deviation of a set of data</li> <li>• Convert data to standard values</li> <li>• Recognize various types of distributions</li> <li>• For a normal distribution, determine the percent of data within a given interval and find percentiles</li> <li>• Recognize different types of sampling procedure</li> <li>• Estimate population characteristics based on samples</li> <li>• Use a sample proportion to find a confidence interval for a corresponding population proportion</li> </ul>

**Unit 4: Sequences and Series**

<b>Topics</b>	Arithmetic and geometric sequences and series, explicit and recursive definitions of sequences and series, limits of sequences and series.
<b>Competencies</b>	<p>8. Identifies, extends, and generalizes a variety of patterns.</p> <p>9. Computes partial sums of infinite arithmetic sequences, determines when an infinite geometric series converges, and finds its sum. Works between recursive and explicit representations.</p> <p>10. Identifies arithmetic and geometric sequences and finds the <math>n</math>th term; then Uses the generalization to find a specific term.</p>
<b>Knowledge/Skills</b>	<ul style="list-style-type: none"> <li>• Identify arithmetic and geometric sequence and find a formula for their <math>n</math>th term</li> <li>• Use sequences defined recursively to solve problems</li> <li>• Find the sum of the first <math>n</math> terms of arithmetic or geometric sequences</li> <li>• Find or estimate the limit of an infinite sequence or determine if the limit does not exist</li> <li>• Find the sum of an infinite geometric series</li> </ul>

	<ul style="list-style-type: none"> <li>• Represent series using sigma notation</li> <li>• Use mathematical induction to prove that a statement is true</li> </ul>

**Unit 5: Linear and Quadratic Functions**

<b>Topics</b>	Functions and relations, finding roots of polynomial functions, maximums and minimums of functions
<b>Competencies</b>	11. Demonstrates conceptual understanding of linear and nonlinear functions and relations
<b>Knowledge/Skills</b>	<ul style="list-style-type: none"> <li>• Graph linear and quadratic equations on the coordinate plane</li> <li>• Find points of intersections of lines and parabolas</li> <li>• Use slope, distance and midpoint formulas to verify properties of quadrilaterals</li> <li>• Find equations of perpendicular and parallel lines</li> <li>• Find equations of medians, altitudes, perpendicular bisectors of the sides of triangles</li> <li>• Use linear and quadratic models to solve real world problems</li> <li>• Find real and imaginary roots of quadratic Equations</li> </ul> <p>Polynomial Functions</p> <ul style="list-style-type: none"> <li>• Solve polynomial equations using the graphing calculator as well as synthetic division techniques</li> <li>• Graph polynomial functions on the coordinate plane</li> <li>• Determine the equation of a polynomial function from its graph</li> <li>• Find maximum and minimum points of cubic and quadratic functions</li> <li>• Solve problems from business, science and manufacturing using maximum and minimum values</li> </ul> <p>Functions</p> <ul style="list-style-type: none"> <li>• Determine when a relation is a function</li> <li>• Determine domain, range and zeros of a function</li> <li>• Graph a function in two variables on the coordinate plane</li> <li>• Explore the relationship between the rule for a function and the graph of that function</li> <li>• Sketch graphs of functions using symmetry and transformations such as stretching,</li> </ul>

	shrinking, reflecting, and translating <ul style="list-style-type: none"> <li>• Determine periodicity and amplitude of functions</li> <li>• Find and graph the inverse of a function if it exists</li> <li>• Determine a minimum or maximum value of a function if one exists</li> </ul>

### Unit 6: Exponents and Radicals

<b>Topics</b>	Exponential equations and logarithmic equations
<b>Competencies</b>	13. Solves equations involving exponential and logarithmic expressions, graphs and interprets the solutions.
<b>Knowledge/Skills</b>	<ul style="list-style-type: none"> <li>• Simplify exponential expressions involving integral and rational exponents</li> <li>• Solve problems involving exponential growth and decay</li> <li>• Solve problems such as compound interest using exponential equations</li> <li>• Simplify logarithmic expressions</li> <li>• Solve exponential equations using logarithms</li> <li>• Apply the laws of logarithms to expand or condense a logarithmic expression</li> <li>• Use the change of base rule for logarithms</li> <li>• Solve problems using natural log</li> </ul>

### Unit 7: Triangle Trigonometry

<b>Topics</b>	Law of sines and cosines, solving oblique triangles
<b>Competencies</b>	
<b>Knowledge/Skills</b>	<ul style="list-style-type: none"> <li>• Use trigonometry to solve problems involving right triangles</li> <li>• Use trigonometry to find the area of a triangle</li> <li>• Use the laws of sines and cosines to solve oblique triangles</li> <li>• Use the laws of sines and cosines solve problems involving navigation and surveying</li> </ul>

<b>Unit 8: Trigonometric Functions</b>	
<b>Topics</b>	Graphing trigonometric functions, inverse trigonometric functions
<b>Competencies</b>	<ol style="list-style-type: none"> <li>4. Derives and uses formulas for length of arcs and area of sectors and area of segments of circles.</li> <li>5. Uses radian measures appropriately and converts between radian measure and degree measure; and understands why radian measure is useful.</li> <li>6. Solves problems using analytic geometry and circular trigonometry.</li> </ol>
<b>Knowledge/Skills</b>	<ul style="list-style-type: none"> <li>• Determine angle measure using degree measure and radian measure</li> <li>• Find arc length and area of sectors using both radian measure and degrees</li> <li>• Use the definitions of the six trigonometric functions to find their values and to graph</li> <li>• Find all values and principal values of the inverse trigonometric functions</li> <li>• Graph the six trigonometric functions and their inverses</li> </ul>
<b>Unit 9: Trigonometric Equations and Applications</b>	
<b>Topics</b>	Verifying trigonometric identities and solving trigonometric functions
<b>Competencies</b>	12. Solves equations and verifies/proves identities.
<b>Knowledge/Skills</b>	<ul style="list-style-type: none"> <li>• Derive and apply trigonometric formulas</li> <li>• Use trigonometric functions to model periodic behavior</li> <li>• Simplify trigonometric expressions</li> <li>• Prove trigonometric identities</li> <li>• Solve trigonometric equations using identities</li> </ul>
<b>Unit 10: Trigonometric Addition Formulas</b>	

<b>Topics</b>	Verifying trigonometric identities and solving trigonometric functions with sums, differences and multiple angles
<b>Competencies</b>	12. Solves equations and verifies/proves identities.
<b>Knowledge/Skills</b>	<ul style="list-style-type: none"> <li>• Derive and apply addition formulas for sine, cosine and tangent</li> <li>• Derive and apply double-angle and half-angle formulas for sine, cosine and tangent</li> <li>• Use addition identities to solve trigonometric equations</li> </ul>

### Unit 11: Polar Coordinates and Complex Numbers

<b>Topics</b>	Polar equations and complex numbers
<b>Competencies</b>	<ol style="list-style-type: none"> <li>1. Solves problems involving complex numbers.</li> <li>2. Manipulates complex numbers using rectangular and polar coordinates.</li> <li>3. Finds the nth roots of complex numbers using De Moivre's Theorem.</li> </ol>
<b>Knowledge/Skills</b>	<ul style="list-style-type: none"> <li>• Graph polar equations</li> <li>• Write complex numbers in polar form</li> <li>• Find products and quotients in polar form</li> <li>• Use DeMoivre's theorem to find powers of complex numbers</li> <li>• Find the roots of complex numbers</li> <li>• Convert from rectangular coordinates to polar coordinates and vice versa</li> </ul>

### Unit 12: Vectors and Determinants

<b>Topics</b>	Vector operations, parametric equations
<b>Competencies</b>	14. Uses matrices and determinants to solve systems of equations.
<b>Knowledge/Skills</b>	<ul style="list-style-type: none"> <li>• Perform basic vector operations with vectors</li> <li>• Use vector and parametric equations to describe motion in a plane</li> <li>• Define and apply the dot product and cross</li> </ul>

	product to vectors <ul style="list-style-type: none"> <li>• Sketch planes and find the equations of planes</li> <li>• Define and evaluate determinants</li> <li>• Use determinants to solve algebraic and geometric problems</li> <li>• Define and apply the cross product</li> </ul>
<b>Unit 13: Analytic Geometry</b>	
<b>Topics</b>	Analytic geometry, conic sections
<b>Competencies</b>	6. Solves problems using analytic geometry and circular trigonometry.
<b>Knowledge/Skills</b>	<ul style="list-style-type: none"> <li>• Find the equations and use the equations to graph circles, parabolas, ellipses and hyperbolas</li> <li>• Find points of intersection between lines or pairs of conics using both algebraic and geometric methods</li> <li>• Solve systems of second degree equations</li> <li>• Define conic sections using eccentricity</li> <li>• Identify the graphs of second-degree equations using the coefficients of the equation</li> </ul>